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ON STOCHASTIC OPTIMAL REINSURANCE AND INVESTMENT STRATEGIES FOR THE SURPLUS

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Abstract. When we consider a life insurance company that sells a large number of continuous T-year term life insurance policies, it is important to find an optimal strategy which maximizes the surplus of the insurance company at time T. The purpose of this paper is to give an explicit expression for the optimal reinsurance and investment strategy which maximizes the expected exponential utility of the final value of the surplus at the end of T-th year. To do this we solve the corresponding Hamilton-Jacobi-Bellman equation.

1. Introduction

By the surplus we mean the excess of some initial fund plus premiums collected over claims paid. The insurance company's risk will be reduced through reinsurance, while in addition the company invests its surplus in a financial market. Assume that a life insurance company writes continuous T-year term life insurance policies for a large number of policyholders, which are defined on the interval $[0, T]$ and provide a payment at the moment of death if the death occurs in $[0, T]$. Two of fundamental aims that the insurance company pursues are to minimize the ruin probability of the company and to maximize the expected utility of the final surplus at the end of the T-th year.

In this paper we assume that, in the case of no reinsurance and no investment, the surplus process $(F(t))_{t\in[0,T]}$ is described by the following

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diffusion form:

$$(1.1) \qquad \begin{cases} dF(t) = \mu dt + \sigma_0 dB_0(t) \\ F(0) = F, \end{cases}$$

where the second term of the right hand side is the stochastic integral w.r.t. a 1-dimensional standard Brownian motion $(B_0(t))_{t\geq 0}$. The constant $F > 0$ is the initial surplus, while the constants $\mu > 0$ and $\sigma_0 > 0$ are the exogenous parameters. This type of model for surplus was treated by Hipp and Plum [3], Hϕjgaad and Taksar [4], Luo, Taksar and Tsoi [5] and Schmidli [7]. Two major types of mathematical models for the surplus are the classical Cramer-Lundberg model and a linear diffusion process described by (1.1). It is known that the latter is given by a diffusion approximation for the former defined by

$$F(t) = F + pt - \sum_{k=1}^{N(t)} X_k$$

where $p > 0$, $F \geq 0$, and $(N(t))_{t\geq 0}$ is a Poisson process of the incoming claims and $X_1, X_2, \cdots$ are i.i.d. random variables representing the size of the successive claims (see [5] or [9]). The surplus described by (1.1) may be used when an insurance company deals with a large number of policyholders where an individual claim is relatively small compared with the size of the surplus.

The proportional reinsurance level at time $t \in [0, T]$ will be associated with the value $1 - u(t)$, where $0 \leq u(t) \leq 1$ is called the risk exposure. If the cedent choose the risk exposure $u(t)$, then the cedent have to pay $100u(t)\%$ of each claim while the rest $100(1 - u(t))\%$ of the claim will be paid by the reinsurer. To purchase this reinsurance, the cedent pays part of the premiums to the reinsurer at the rate of $(1 - u(t))\lambda$ where $\lambda \geq \mu$. Then the corresponding surplus process $(F(t))_{t\in[0,T]}$ is given by

$$(1.2) \qquad \begin{cases} dF(t) = \{\mu - (1 - u(t))\lambda\}dt + u(t)\sigma_0 dB_0(t) \\ F(0) = F. \end{cases}$$

The constants μ and λ can be regarded as the safety loading of the cedent and reinsurer, respectively.

In addition, we assume that all of the surplus is invested in a financial market which consists of two stocks, named X_I and X_{II}, whose prices are given by the following linear stochastic differential equations:

$$\begin{cases} dX_i(t) = a_i X_i(t)dt + \sigma_i X_i(t) dB_i(t) \\ X_i(0) = x_i, \ \ i = 1, 2 \end{cases} \tag{1.3}$$

where a_i and σ_i, $i = 1, 2$, are the constants satisfying $a_1 \leq a_2$ and $\sigma_1 < \sigma_2$, and $(B_i(t))_{t\geq 0}$, $i = 1, 2$, are independent standard Brownian motions independent of $(B_0(t))_{t\geq 0}$. We can say that the stock X_{II} is more risky stock than the stock X_I. We denote by $v(t)$ the proportion invested in more risky stock X_{II} at time $t \in [0, T]$. We disallow leverage and short-sales, which restrict $v(t)$ to be in 0 and 1, i.e. $0 \leq v(t) \leq 1$. Therefore, at any time $0 \leq t < T$, a nominal amount $F(t)(1 - v(t))$ is allocated to the stock X_I. We treat the risk exposure $u(t)$ and the proportion $v(t)$ of the surplus at time t being invested in more risky stock X_{II} as control parameters. Then the surplus process $(F(t))_{t\in[0,T]}$ is given by the following linear stochastic differential equations:

$$\begin{cases} dF(t) = \{F(t)[v(t)a_2 + (1 - v(t))a_1] + \mu - (1 - u(t))\lambda\}dt \\ \qquad\qquad + u(t)\sigma_0 dB_0(t) + F(t)\sigma_1(1 - v(t))dB_1(t) \\ \qquad\qquad + F(t)\sigma_2 v(t) dB_2(t) \\ F(0) = F. \end{cases} \tag{1.4}$$

Given a strategy $(u(\cdot), v(\cdot))$, the solution $(F^{u,v}(t))_{t\in[0,T]}$ is called the surplus process corresponding to $(u(\cdot), v(\cdot))$. In the case that $v(t) \equiv 1$ in (1.4), i.e., all of the surplus is invested in the stock X_{II} only, Taksar and Markussen [8] gave an explicit expression for the optimal reinsurance policy which minimizes the ruin probability of cedent. And Luo, Taksar and Tsoi [5] extended results in [8] to the case that $\sigma_1 = 0$ in (1.4), i.e., X_I is a riskless asset. In the case that $\sigma_1 = 0$ and $u(t) \equiv 0$ in (1.4), i.e., there is no reinsurance, Devolder et al. [2] found an explicit expression for the optimal asset allocation which maximizes the expected utility of the final annuity fund at retirement and at the end of the period after retirement.

In this paper we find an explicit expression for the optimal strategy $(u^*(\cdot), v^*(\cdot))$ which maximizes the expected exponential utility of the final value of the surplus process given by the stochastic differential equation (1.4). To do this we solve the corresponding Hamilton-Jacobi-Bellman (HJB) equation.

The structure of the paper is as follows. In Section 2 we formulate our problem and give main results. All proofs are based on stochastic optimal control theory (see Björk [1] or Øksendal [6]). They are presented in Section 3.

2. Formulation of the problem and main results

In this section we formulate our problem and give main results. Let $(\Omega, \mathcal{F}, P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ on which three independent $\{\mathcal{F}_t\}_{t\geq 0}$-adapted standard Brownian motions $(B_i(t))_{t\geq 0}$, $i = 0, 1, 2$, are defined. A control $(u(\cdot), v(\cdot))$ is said to be admissible if $u(\cdot)$ and $v(\cdot)$ are $\{\mathcal{F}_t\}_{t\geq 0}$-adapted processes satisfying $0 \leq u(t), v(t) \leq 1$ for all $t \in [0, T]$. The set of all admissible controls is denoted by $\mathcal{A}$.

We use an exponential utility function of the form:

$$U(x) = -\frac{1}{c}e^{-cx}, \ c > 0. \tag{2.1}$$

Since $U'(x) > 0$ and $U''(x) < 0$ for all $x \in [0, \infty)$, $U(x)$ may serve as the utility function of a risk-averse individual. Put

$$J^{u,v}(t, x) = E[U(F^{u,v}(T)) \mid F^{u,v}(t) = x], \ (t, x) \in [0, T] \times \mathbb{R}^1, \tag{2.2}$$

where $E[X|A]$ is the conditional expectation of a random variable X given an event A. Our optimal control problem is to find the optimal value function

$$W(t, x) = \sup_{(u,v)\in\mathcal{A}} J^{u,v}(t, x) \tag{2.3}$$

and the optimal strategy $(u^*(\cdot), v^*(\cdot))$ such that

$$J^{u^*,v^*}(t, x) = W(t, x). \tag{2.4}$$

More we will give an explicit expression of $(u^*(t), v^*(t))$. The following two theorems are essential to solve our problem. The proofs are standard and can be found in Chapter 14 of [1] or Chapter 11 of [6].

THEOREM 2.1.**(HJB equation)** *Assume that $W(t, x)$ defined by (2.3) is twice continuously differentiable on $(0, \infty)$, i.e., $\in C^{1,2}$. Then*

$W(t,x)$ satisfies the following HJB equation:

$$\left\{\begin{array}{l}\sup\limits_{(u,v)\in\mathcal{A}} L^{u,v}W(t,x)=0\\ W(T,x)=U(x)\end{array}\right. \tag{2.5}$$

for all $(t,x)\in[0,T)\times\mathbb{R}^1$, where $L^{u,v}$ is the infinitesimal generator corresponding to the diffusion process defined by the stochastic differential equation (1.4), i.e.,

(2.6)

$$\begin{aligned}L^{u,v} &= \frac{\partial}{\partial t}+\{x[v(t)a_2+(1-v(t))a_1]+\mu-(1-u(t))\lambda\}\,\frac{\partial}{\partial x}\\ &\quad+\left\{\frac{1}{2}u(t)^2\sigma_0^2+\frac{1}{2}x^2\left[(1-v(t))^2\sigma_1^2+v(t)^2\sigma_2^2\right]\right\}\frac{\partial^2}{\partial x^2}\\ &= \frac{\partial}{\partial t}+\{x[a_1+(a_2-a_1)v(t)]+\mu-(1-u(t))\lambda\}\,\frac{\partial}{\partial x}\\ &\quad+\left\{\frac{1}{2}u(t)^2\sigma_0^2+\frac{1}{2}x^2\left[\sigma_1^2-2\sigma_1^2v(t)+(\sigma_1^2+\sigma_2^2)v(t)^2\right]\right\}\frac{\partial^2}{\partial x^2}.\end{aligned}$$

THEOREM 2.2. **(Verification theorem)** *Let $H(t,x)\in C^{1,2}$ be a solution of the HJB equation (2.5). Then the value function $W(t,x)$ to the control problem (2.3) is given by*

$$W(t,x)=H(t,x).$$

Moreover if for some control $(\bar{u}(\cdot),\bar{v}(\cdot))$

$$L^{\bar{u},\bar{v}}H(t,x)=0$$

for all $(t,x)\in[0,T)\times\mathbb{R}^1$, then it holds

$$H(t,x)=J^{\bar{u},\bar{v}}(t,x).$$

In this case $(\bar{u}(t),\bar{v}(t))=(u^*(t),v^*(t))$ and $J^{\bar{u},\bar{v}}(t,x)=J^{u^*,v^*}(t,x)$.

The following theorem is our main result.

THEOREM 2.3. *The optimal value function $W(t,x)$ defined by (2.3) and the optimal strategy $(u^*(t),v^*(t))$ are given by*

$$W(t,x)=-\frac{1}{c}\exp\{-c[J(t)+H(t)(x-K(t))]\}, \tag{2.7}$$

(2.8) $$u^*(t) = \left(\frac{\lambda}{c\sigma_0^2}\right) e^{A(t-T)}$$

and

(2.9) $$v^*(t) = \left(\frac{1}{cx}\right)\left(\frac{a_2 - a_1}{\sigma_1^2 + \sigma_2^2}\right) e^{A(t-T)} + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2},$$

where

$$\begin{aligned} J(t) &= -\left(\frac{(a_2 - a_1)^2}{2c(\sigma_1^2 + \sigma_2^2)} + \frac{\lambda^2}{\sigma_0^2}\right)(t - T), \\ H(t) &= e^{-A(t-T)}, \\ K(t) &= -\frac{B}{2A}\left(1 - e^{-A(t-T)}\right) - \frac{\mu - \lambda}{A}\left(1 - e^{A(t-T)}\right). \end{aligned}$$

Here A and B are constants defined by

$$A = a_1 + \frac{(a_2 - a_1)\sigma_1^2}{\sigma_1^2 + \sigma_2^2}, \quad B = \frac{cx^2\sigma_1^2}{2}\left(1 - \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\right).$$

3. Proofs

In this section we prove Theorem 2.3. By Theorem 2.1, the optimal value function $W(t,x)$ defined by (2.3) satisfies the following HJB equation

$$\begin{cases} \sup_{(u,v)\in\mathcal{A}} L^{u,v}W(t,x) = 0 \\ W(T,x) = U(x) \end{cases}$$

for all $(t,x) \in [0,T) \times \mathbb{R}^1$. Define

$$\begin{aligned} \eta(u,v) &= L^{u,v}W \\ &= \frac{\partial W}{\partial t} + \{x[a_1 + (a_2 - a_1)v(t)] + \mu - (1 - u(t))\lambda\}\frac{\partial W}{\partial x} \\ &\quad + \left\{\frac{1}{2}u(t)^2\sigma_0^2 + \frac{1}{2}x^2\left[\sigma_1^2 - 2\sigma_1^2 v(t) + (\sigma_1^2 + \sigma_2^2)v(t)^2\right]\right\}\frac{\partial^2 W}{\partial x^2} \end{aligned}$$

for any control $(u(\cdot), v(\cdot)) \in \mathcal{A}$, and assume that $(\bar{u}(\cdot), \bar{v}(\cdot)) \in \mathcal{A}$ satisfies

$$\eta(\bar{u}, \bar{v}) = 0 \tag{3.1}$$

$$\frac{\partial \eta}{\partial u}(\bar{u}, \bar{v}) = 0 \tag{3.2}$$

$$\frac{\partial \eta}{\partial v}(\bar{u}, \bar{v}) = 0 \tag{3.3}$$

$$\frac{\partial^2 \eta}{\partial u \partial v}(\bar{u}, \bar{v}) - \frac{\partial \eta}{\partial u}(\bar{u}, \bar{v})\frac{\partial \eta}{\partial v}(\bar{u}, \bar{v}) = 0 \tag{3.4}$$

$$\frac{\partial^2 \eta}{\partial u^2}(\bar{u}, \bar{v}) = 0. \tag{3.5}$$

Then by Theorem 2.2, we have $(\bar{u}, \bar{v}) = (u^*, v^*)$. So from now we will prove (2.7) $\sim$ (2.9) by using (3.1) $\sim$ (3.3), and confirm also that (u^*, v^*) expressed by (2.8) and (2.9) satisfies (3.4) and (3.5). From (3.2) we have

$$0 = \lambda \frac{\partial W}{\partial x} + \sigma_0^2 u^*(t) \frac{\partial^2 W}{\partial x^2}$$

and hence

$$u^*(t) = -\frac{\lambda \frac{\partial W}{\partial x}}{\sigma_0^2 \frac{\partial^2 W}{\partial x^2}}. \tag{3.6}$$

And from (3.3) we have

$$0 = (a_2 - a_1)x \frac{\partial W}{\partial x} + \left[(\sigma_1^2 + \sigma_2^2)v^*(t) - \sigma_1^2\right] x^2 \frac{\partial^2 W}{\partial x^2}$$

and hence

$$v^*(t) = -\frac{\frac{\partial W}{\partial x}}{x \frac{\partial^2 W}{\partial x^2}} \cdot \frac{a_2 - a_1}{\sigma_1^2 + \sigma_2^2} + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}. \tag{3.7}$$

Inserting (3.6) and (3.7) into (3.1), we obtain the following partial differential equation for the optimal value function W:

$$\begin{aligned} 0 = \frac{\partial W}{\partial t} &+ \left(a_1 x + \mu - \lambda + \frac{(a_2 - a_1)\sigma_1^2 x}{\sigma_1^2 + \sigma_2^2} \right) \frac{\partial W}{\partial x} \\ &+ \frac{x^2 \sigma_1^2}{2} \left(1 - \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \right) \frac{\partial^2 W}{\partial x^2} \\ &- \frac{1}{2} \left(\frac{(a_2 - a_1)^2 \sigma_1^2 x}{\sigma_1^2 + \sigma_2^2} + \frac{\lambda^2}{\sigma_0^2} \right) \frac{\left(\frac{\partial W}{\partial x}\right)^2}{\frac{\partial^2 W}{\partial x^2}} \end{aligned} \tag{3.8}$$

with the boundary condition

$$W(T,x)=U(x)=-\frac{1}{c}e^{-cx}. \tag{3.9}$$

We try to find a solution of (3.8) and (3.9) with the following structure:

$$W(t,x)=-\frac{1}{c}\exp\{-c[J(t)+H(t)(x-K(t))]\} \tag{3.10}$$

with $J(T)=0$, $H(T)=1$ and $K(T)=0$ as terminal conditions. Then it holds

$$\frac{\partial W}{\partial t}=[J'(t)+H'(t)(x+K(t))-H(t)K'(t)]\exp\{-c[J(t)+H(t)(x-K(t)]\}.$$

Put

$$w(t,x)=J(t)+H(t)(x-K(t)).$$

Then we get

$$\frac{\partial W}{\partial t}=[J'(t)+H'(t)(x+K(t))-H(t)K'(t)]e^{-cw(t,x)}. \tag{3.11}$$

Similarly we get

$$\frac{\partial W}{\partial x}=H(t)e^{-cw(t,x)} \tag{3.12}$$

and

$$\frac{\partial^2 W}{\partial x^2}=-cH^2(t)e^{-cw(t,x)}. \tag{3.13}$$

Introducing (3.11), (3.12) and (3.13) in (3.8), it holds

$$\begin{aligned}0=&[J'(t)+H'(t)(x+K(t))-H(t)K'(t)]e^{-cw(t,x)}\\&+\left(a_1x+\mu-\lambda+\frac{(a_2-a_1)\sigma_1^2x}{\sigma_1^2+\sigma_2^2}\right)H(t)e^{-cw(t,x)}\\&+\frac{x^2\sigma_1^2}{2}\left(1-\frac{\sigma_1^2}{\sigma_1^2+\sigma_2^2}\right)\left(-cH^2(t)e^{-cw(t,x)}\right)\\&+\frac{1}{2}\left(\frac{(a_2-a_1)^2\sigma_1^2x}{\sigma_1^2+\sigma_2^2}+\frac{\lambda^2}{\sigma_0^2}\right)\frac{H^2(t)e^{-2cw(t,x)}}{-cH^2(t)e^{-cw(t,x)}}\end{aligned}$$

or

$$\begin{aligned}0 =& J'(t) + \frac{(a_2 - a_1)^2}{2c(\sigma_1^2 + \sigma_2^2)} + \frac{\lambda^2}{\sigma_0^2} \\ &+ xH'(t) + \left(a_1 + \frac{(a_2 - a_1)\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \right) xH(t) \\ &+ H(t)K'(t) + H'(t) - (\mu - \lambda)H(t) + \frac{cH^2(t)x^2\sigma_1^2}{2} \left(1 - \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \right).\end{aligned}$$

We can split this equation into three ordinary differential equations as follows;

Equation (J):

$$\begin{cases} J'(t) + \dfrac{(a_2 - a_1)^2}{2c(\sigma_1^2 + \sigma_2^2)} + \dfrac{\lambda^2}{\sigma_0^2} = 0 \\ J(T) = 0. \end{cases}$$

Equation (H):

$$\begin{cases} H'(t) + \left(a_1 + \dfrac{(a_2 - a_1)\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \right) H(t) = 0 \\ H(T) = 1. \end{cases}$$

Equation (K):

$$\begin{cases} H(t)K'(t) + H'(t) - (\mu - \lambda)H(t) + \dfrac{cH^2(t)x^2\sigma_1^2}{2} \left(1 - \dfrac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \right) = 0 \\ K(T) = 0. \end{cases}$$

The solutions of Equation (J) and (H) are given by

$$J(t) = \left(\frac{(a_2 - a_1)^2}{2c(\sigma_1^2 + \sigma_2^2)} + \frac{\lambda^2}{\sigma_0^2} \right) (T - t) \tag{3.14}$$

and

$$H(t) = e^{-A(T-t)}, \tag{3.15}$$

respectively, where

$$A = a_1 + \frac{(a_2 - a_1)\sigma_1^2}{\sigma_1^2 + \sigma_2^2}.$$

Substituting (3.15) in Equation (K), it becomes

$$K'(t) - AK(t) - (\mu - \lambda) + Be^{-A(T-t)} = 0,$$

where

$$B = \frac{cx^2\sigma_1^2}{2}\left(1 - \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}\right).$$

The solution of this equation satisfying $K(T) = 0$ is given by

$$K(t) = -\frac{B}{2A}\left(1 - e^{-A(t-T)}\right) - \frac{\mu - \lambda}{A}\left(1 - e^{A(t-T)}\right). \tag{3.16}$$

Inserting (3.13), (3.14) and (3.15) in (3.10), we can see that the optimal value function $W(t, x)$ is given by (2.8).
Clearly (3.4) and (3.5) are fulfilled. In fact, from the definition of $\eta(u, v)$, we have

$$\begin{aligned}\frac{\partial^2\eta}{\partial u^2}(u^*, v^*) &= \sigma_0^2\frac{\partial^2 W}{\partial x^2}\\ &= \sigma_0^2\left(-cH^2(t)\right)e^{-cw(t,x)}\\ &= -\sigma_0^2 cH^2(t)e^{-cw(t,x)}\\ &< 0\end{aligned}$$

and

$$\begin{aligned}\left(\frac{\partial^2\eta}{\partial u\partial v} - \frac{\partial\eta}{\partial u}\cdot\frac{\partial\eta}{\partial v}\right)(u^*, v^*) &= 0 - \sigma_0^2\left[x^2(\sigma_1^2 + \sigma_2^2)\right]\left(\frac{\partial^2 W}{\partial x^2}\right)^2\\ &= -\sigma_0^2\left[x^2(\sigma_1^2 + \sigma_2^2)\right]c^2H^4(t)e^{-2cw(t,x)}\\ &< 0.\end{aligned}$$

Inserting (3.12) and (3.13) into (3.6), we get

$$\begin{aligned}u^*(t) &= -\frac{\lambda\frac{\partial W}{\partial x}}{\sigma_0^2\frac{\partial^2 W}{\partial x^2}} = \left(-\frac{\lambda}{\sigma_0^2}\right)\frac{H(t)e^{-cw(t,x)}}{-cH^2(t)e^{-cw(t,x)}}\\ &= \left(\frac{\lambda}{\sigma_0^2}\right)\frac{1}{cH(t)} = \left(\frac{\lambda}{c\sigma_0^2}\right)e^{A(t-T)}.\end{aligned}$$

Finally inserting (3.12) and (3.13) into (3.7), we get

$$\begin{aligned} v^*(t) &= -\frac{\frac{\partial W}{\partial x}}{x\frac{\partial^2 W}{\partial x^2}} \cdot \frac{a_2 - a_1}{\sigma_1^2 + \sigma_2^2} + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \\ &= \frac{H(t)e^{-cw(t,x)}}{-cH^2(t)e^{-cw(t,x)}} \cdot \frac{a_2 - a_1}{\sigma_1^2 + \sigma_2^2} + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2} \\ &= \left(\frac{1}{cx}\right)\left(\frac{a_2 - a_1}{\sigma_1^2 + \sigma_2^2}\right) e^{A(t-T)} + \frac{\sigma_1^2}{\sigma_1^2 + \sigma_2^2}. \end{aligned}$$

The proof of Theorem 2.3 is complete.

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