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REMARKS ON INTERVAL-VALUED FUZZY MINIMAL PRECONTINUOUS MAPPINGS AND INTERVAL-VALUED FUZZY MINIMAL PREOPEN MAPPINGS

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Abstract. In [5], we introduced the concepts of IVF m-preopen sets and IVF m-precontinuous mappings on interval-valued fuzzy minimal spaces. In this paper, we introduce the concept of IVF m-preopen mapping and investigate characterizations for IVF m-precontinuous mappings and IVF m-preopen mappings.

1. Introduction

Zadeh [7] introduced the concept of fuzzy set and several researchers were concerned about the generalizations of the concept of fuzzy sets, intuitionistic fuzzy sets [1] and interval-valued fuzzy sets [3]. Alimohammady and Roohi [2] introduced fuzzy minimal structures and fuzzy minimal spaces and some results are given. In [4], Min introduced the concepts of IVF minimal structures and IVF m-continuous mappings which are generalizations of IVF topologies and IVF continuous mappings [6], respectively. In [5], Min et al. introduced the concepts of IVF m-preopen sets and IVF m-precontinuous mappings on interval-valued fuzzy minimal spaces. In this paper, we introduce the concept of IVF m-preopen mapping and investigate characterizations for IVF m-precontinuous mappings and IVF m-preopen mappings.

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2. Preliminaries

Let $D[0,1]$ be the set of all closed subintervals of the interval $[0,1]$. The elements of $D[0,1]$ are generally denoted by capital letters $M, N, \cdots$ and note that $M = [M^L, M^U]$, where M^L and M^U are the lower and the upper end points, respectively. Especially, we denote $\mathbf{0} = [0,0], \mathbf{1} = [1,1]$, and $\mathbf{a} = [a,a]$ for $a \in (0,1)$. We also note that

(1) $(\forall M, N \in D[0,1])(M = N \Leftrightarrow M^L = N^L, M^U = N^U)$.

(2) $(\forall M, N \in D[0,1])(M \leq N \Leftrightarrow M^L \leq N^L, M^U \leq N^U)$.

For every $M \in D[0,1]$, the complement of M, denoted by M^c, is defined by $M^c = 1 - M = [1 - M^U, 1 - M^L]$.

Let X be a nonempty set. A mapping $A : X \to D[0,1]$ is called an interval-valued fuzzy set (simply, IVF set) in X. For each $x \in X$, $A(x)$ is a closed interval whose lower and upper end points are denoted by $A(x)^L$ and $A(x)^U$, respectively. For any $[a,b] \in D[0,1]$, the IVF set whose value is the interval $[a,b]$ for all $x \in X$ is denoted by $\widetilde{[a,b]}$. In particular, for any $c \in [a,b]$, the IVF set whose value is $\mathbf{c} = [c,c]$ for all $x \in X$ is denoted by simply $\widetilde{a}$. For a point $p \in X$ and for $[a,b] \in D[0,1]$ with $b > 0$, the IVF set which takes the value $[a,b]$ at p and $\mathbf{0}$ elsewhere in X is called an interval-valued fuzzy point (simply, IVF point) and is denoted by $[a,b]_p$. In particular, if $b = a$, then it is also denoted by a_p. We denote the set of all IVF sets by IVF(X). An IVF point M_x, where $M \in D[0,1]$, is said to belong to an IVF set A in X, denoted by $M_x \widetilde{\in} A$, if $A(x)^L \geq M^L$ and $A(x)^U \geq M^U$. In [6], it has been shown that $A = \cup\{M_x : M_x \widetilde{\in} A\}$. For every $A, B \in IVF(X)$,

$$A = B \Leftrightarrow (\forall x \in X)([A(x)]^L = [B(x)]^L \text{ and } [A(x)]^U = [B(x)]^U),$$

$$A \subseteq B \Leftrightarrow (\forall x \in X)([A(x)]^L \subseteq [B(x)]^L \text{ and } [A(x)]^U \subseteq [B(x)]^U).$$

The complement A^c of A is defined by

$$[A^c(x)]^L = 1 - [A(x)]^U \text{ and } [A^c(x)]^U = 1 - [A(x)]^L$$

for all $x \in X$.

For a family of IVF sets $\{A_i : i \in J\}$ where J is an index set, the union $G = \cup_{i \in J} A_i$ and $F = \cap_{i \in J} A_i$ are defined by

$$(\forall x \in X)([G(x)]^L = \sup_{i \in J}[A_i(x)]^L, [G(x)]^U = \sup_{i \in J}[A_i(x)]^U),$$

$$(\forall x \in X)([F(x)]^L = \inf_{i \in J}[A_i(x)]^L, [F(x)]^U = \inf_{i \in J}[A_i(x)]^U),$$

respectively.

Let $f : X \to Y$ be a mapping and let A be an IVF set in X. Then the image of A under f, denoted by $f(A)$, is defined as follows

$$[f(A)(y)]^L = \begin{cases} \sup\limits_{f(x)=y} [A(x)]^L, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise }, \end{cases}$$

$$[f(A)(y)]^U = \begin{cases} \sup\limits_{f(x)=y} [A(x)]^U, & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise }, \end{cases}$$

for all $y \in Y$.

Let B be an IVF set in Y. Then the inverse image of B under f, denoted by $f^{-1}(B)$, is defined as follows

$$(\forall x \in X)([f^{-1}(B)(x)]^L = [B(f(x))]^L, [f^{-1}(B)(x)]^U = [B(f(x))]^U).$$

Definition 2.1 ([6]). A family τ of IVF sets in X is called an *interval-valued fuzzy topology* on X if it satisfies:

(1) $\mathbf{0}, \mathbf{1} \in \tau$.

(2) $A, B \in \tau \Rightarrow A \cap B \in \tau$.

(3) For $i \in J$, $A_i \in \tau \Rightarrow \cup_{i \in J} A_i \in \tau$.

Every member of τ is called an IVF open set. An IVF set A is called an IVF closed set if the complement of A is an IVF open set. And (X, τ) is called an *interval-valued fuzzy topological space.*

Definition 2.2 ([4]). A family $\mathfrak{M}$ of interval-valued fuzzy sets in X is called an *interval-valued fuzzy minimal structure* on X if

$$\mathbf{0}, \mathbf{1} \in \mathfrak{M}.$$

In this case, $(X, \mathfrak{M})$ is called an *interval-valued fuzzy minimal space* (simply, *IVF minimal space*). Every member of $\mathfrak{M}$ is called an IVF m-open set. An IVF set A is called an IVF m-closed set if the complement of A (simply, A^c) is an IVF m-open set.

Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). The IVF minimal-closure and the IVF minimal-interior of A [4], denoted by $mC(A)$ and $mI(A)$, respectively, are defined as

$$mC(A) = \cap\{B \in IVF(X) : B^c \in \mathfrak{M} \text{ and } A \subseteq B\},$$

$$mI(A) = \cup\{B \in IVF(X) : B \in \mathfrak{M} \text{ and } B \subseteq A\}.$$

Theorem 2.3 ([4]). *Let $(X, \mathfrak{M})$ be an IVF minimal space and A, B in IVF(X).*

(1) $mI(A) \subseteq A$ and if A is an IVF m-open set, then $mI(A) = A$.
(2) $A \subseteq mC(A)$ and if A is an IVF m-closed set, then $mC(A) = A$.
(3) If $A \subseteq B$, then $mI(A) \subseteq mI(B)$ and $mC(A) \subseteq mC(B)$.
(4) $mI(A) \cap mI(B) \supseteq mI(A \cap B)$ and $mC(A) \cup mC(B) \subseteq mC(A \cup B)$.
(5) $mI(mI(A)) = mI(A)$ and $mC(mC(A)) = mC(A)$.
(6) $\mathbf{1} - mC(A) = mI(\mathbf{1} - A)$ and $\mathbf{1} - mI(A) = mC(\mathbf{1} - A)$.

3. Main Results

Definition 3.1 ([5]). Let $(X, \mathfrak{M})$ be an IVF minimal space and A in IVF(X). Then an IVF set A is called an IVF m-preopen set in X if

$$A \subseteq mI(mC(A)).$$

An IVF set A is called an *IVF m-preclosed* set if the complement of A is IVF m-preopen. The *pre-closure* and the *pre-interior* of A, denoted by $pmC(A)$ and $pmI(A)$, respectively, are defined as the following:

$$pmC(A) = \cap\{F \in IVF(X) : A \subseteq F, F \text{ is IVF } m\text{-preclosed in } X\}$$

$$pmI(A) = \cup\{U \in IVF(X) : U \subseteq A, U \text{ is IVF } m\text{-preopen in } X\}.$$

Theorem 3.2 ([5]). *Let $(X, \mathfrak{M})$ be an IVF minimal space and $A \in IVF(X)$. Then*

(1) $pmI(A) \subseteq A \subseteq pmC(A)$.
(2) If $A \subseteq B$, then $pmI(A) \subseteq pmI(B)$ and $pmC(A) \subseteq pmC(B)$.
(3) A is IVF m-preopen iff $pmI(A) = A$.
(4) F is IVF m-preclosed iff $pmC(F) = F$.
(6) $pmI(pmI(A)) = pmI(A)$ and $pmC(pmC(A)) = pmC(A)$.
(6) $pmC(\mathbf{1} - A) = \mathbf{1} - pmI(A)$ and $pmI(\mathbf{1} - A) = \mathbf{1} - pmC(A)$.

Definition 3.3 ([5]). Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is said to be *interval-valued fuzzy m-precontinuous* (simply, IVF m-precontinuous) if for each IVF point M_x and each IVF m-open set V containing $f(M_x)$, there exists an IVF m-preopen set U containing M_x such that $f(U) \subseteq V$.

Theorem 3.4. *Let $f : X \to Y$ be a function on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:*

(1) f is IVF m-precontinuous.

(2) $f^{-1}(V) \subseteq mI(mC(f^{-1}(V)))$ *for each IVF* m*-open set* V *in* Y*.*
(3) $mC(mI(f^{-1}(F))) \subseteq f^{-1}(F)$ *for each IVF* m*-closed set* F *in* Y*.*
(4) $f(mC(mI(A))) \subseteq mC(f(A))$ *for* $A \in IVF(X)$*.*
(5) $mC(mI(f^{-1}(B))) \subseteq f^{-1}(mC(B))$ *for* $B \in IVF(Y)$*.*
(6) $f^{-1}(mI(B)) \subseteq mI(mC(f^{-1}(B)))$ *for* $B \in IVF(Y)$*.*

Proof. (1) $\Rightarrow$ (2) Let V be an IVF m-open set in Y and $M_x \widetilde{\in} f^{-1}(V)$. Since f is IVF m-precontinuous, there exists an IVF m-preopen set U containing M_x such that $f(U) \subseteq V$. Since U is IVF m-preopen, we have

$$M_x \widetilde{\in} U \subseteq mI(mC(U)) \subseteq mI(mC(f^{-1}(V))).$$

Hence we have $f^{-1}(V) \subseteq mI(mC(f^{-1}(V)))$.

(2) $\Rightarrow$ (1) let M_x be an IVF point of X and V an IVF m-open set containing $f(M_x)$. Then by hypothesis, we have $f^{-1}(V) \subseteq mI(mC(f^{-1}(V)))$. This implies $f^{-1}(V)$ is an IVF m-preopen set containing M_x. Put $U = f^{-1}(V)$. Then $f(U) \subseteq V$ and so f is IVF m-precontinuous.

(2) $\Leftrightarrow$ (3) It is obvious from Theorem 3.2.

(3) $\Rightarrow$ (4) Let $A \in IVF(X)$. Then since $mC(f(A))$ is an IVF m-closed set in Y, from (3), it follows $mC(mI(A)) \subseteq mC(mI(f^{-1}(f(A)))) \subseteq mC(mI(f^{-1}(mC(f(A))))) \subseteq f^{-1}(mC(f(A)))$.

Hence $f(mC(mI(A))) \subseteq mC(f(A))$.

(4) $\Rightarrow$ (5) Obvious.

(5) $\Rightarrow$ (6) For $B \in IVF(Y)$, from (5), it follows:

$$\begin{aligned} f^{-1}(mI(B)) &= f^{-1}(\mathbf{1} - mC(\mathbf{1} - B)) \\ &= \mathbf{1} - (f^{-1}(mC(\mathbf{1} - B))) \\ &\subseteq \mathbf{1} - mC(mI(f^{-1}(\mathbf{1} - B))) \\ &= mI(mC(f^{-1}(B))). \end{aligned}$$

Hence, we have (6).

(6) $\Rightarrow$ (1) Let V an IVF m-open set in Y. Then by (6), we have $f^{-1}(V) = f^{-1}(mI(V)) \subseteq mI(mC(f^{-1}(V)))$. This implies $f^{-1}(V)$ is an IVF m-preopen set. Hence f is IVF m-precontinuous. □

LEMMA 3.5. *Let* $(X, \mathcal{M}_X)$ *be an IVF minimal space and* $A \in IVF(X)$*. Then*

(1) $mC(mI(A)) \subseteq mC(mI(pmC(A))) \subseteq pmC(A)$*.*
(2) $pmI(A) \subseteq mI(mC(pmI(A))) \subseteq mI(mC(A))$*.*
(3) $mC(mI(A)) = pmC(mI(A))$*.*
(4) $pmI(mC(A)) = mI(mC(A))$*.*

Proof. (1) For $A \in IVF(X)$, since $pmC(A)$ is an IVF m-preclosed set we have $mC(mI(A)) \subseteq mC(mI(pmC(A))) \subseteq pmC(A)$.

(2) It is similar to the proof of (1).

(3) For $A \in IVF(X)$, from (1) and Theorem 3.2, we have

$$mC(mI(mI(A))) \subseteq pmC(mI(A)) \subseteq mC(mI(A)).$$

This implies $mC(mI(A)) = pmC(mI(A))$.

(4) It follows from Theorem 3.2 and (2).

□

From Theorem 3.4 and Lemma 3.5, we have the next corollary.

Corollary 3.6. Let $f : X \to Y$ be a function on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:

(1) f is IVF m-precontinuous.

(2) $f^{-1}(V)$ is an IVF m-preopen set for each IVF m-open set V in Y.

(3) $f^{-1}(F)$ is an IVF m-preclosed set for each IVF m-closed set F in Y.

(4) $f(pmC(A)) \subseteq mC(f(A))$ for $A \in IVF(X)$.

(5) $pmC(f^{-1}(B)) \subseteq f^{-1}(mC(B))$ for $B \in IVF(Y)$.

(6) $f^{-1}(mI(B)) \subseteq pmI(f^{-1}(B))$ for $B \in IVF(Y)$.

Definition 3.7. Let $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$ be two IVF minimal spaces. Then $f : X \to Y$ is said to be *interval-valued fuzzy m-preopen* (simply, IVF m-preopen) if for each IVF m-open set U in X, $f(U)$ is IVF m-preopen.

Theorem 3.8. *Let $f : X \to Y$ be a function on IVF minimal spaces $(X, \mathcal{M}_X)$ and $(Y, \mathcal{M}_Y)$. Then the following statements are equivalent:*

(1) f is IVF m-preopen.

(2) $f(mI(A)) \subseteq pmI(f(A))$ for $A \in IVF(X)$.

(3) $mI(f^{-1}(B)) \subseteq f^{-1}(pmI(B))$ for $B \in IVF(Y)$.

Proof. (1) ⇒ (2) Let $A \in IVF(X)$. Then since f is IVF m-preopen, from Theorem 3.2, it follows

$$f(mI(A)) = pmI(f(mI(A))) \subseteq pmI(f(A))$$

Hence (2) is obtained.

(2) ⇒ (3) For $B \in IVF(Y)$, from (2), it follows

$$f(mI(f^{-1}(B))) \subseteq pmI(f(f^{-1}(B))) \subseteq pmI(B)$$

This implies $mI(f^{-1}(B)) \subseteq f^{-1}(pmI(B))$.

(3) $\Rightarrow$ (1) Let U an IVF m-open set in X. Then from (3), we have $U = mI(U) \subseteq mI(f^{-1}(f(U)))) \subseteq f^{-1}(pmI(f(U)))$. This implies $f(U) \subseteq pmI(f(U))$. By Theorem 3.2, $f(U)$ is IVF m-preopen. Hence f is IVF m-preopen. □

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