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PERIODIC SOLUTIONS FOR THE NONLINEAR HAMILTONIAN SYSTEMS

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Abstract. We show the existence of nonconstant periodic solution for the nonlinear Hamiltonian systems with some nonlinearity. We approach the variational method. We use the critical point theory and the variational linking theory for strongly indefinite functional.

1. Introduction

Let $L^2(S^1, R^{2n})$ denote the set of $2n$-tuples of the square integrable 2π periodic functions and choose $z \in L^2(S^1, R^{2n})$. Let $H : R^{2n} \to R \in C^1$ with $H(0, \dots, 0) = 0$ and H_z be its gradient. In this paper we investigate the existence of the nonconstant periodic solution of the nonlinear Hamiltonian system

$$\dot{z} = J(H_z(z)), \tag{1.1}$$

where $z \in L^2(S^1, R^{2n})$, $\dot{z} = \frac{dz}{dt}$, $J = \begin{pmatrix} 0 & -I_n \\ I_n & 0 \end{pmatrix}$, I_n is the n dimensional identity matrix. Let $a \cdot b$ and $| \cdot |$ denote the usual inner product and norm on R^{2n}. Let us set $z = (p, q)$, $p = (z_1, \cdots, z_n)$, $q = (z_{n+1}, \cdots, z_{2n}) \in R^n$. Then (1.1) can be rewritten as

$$\begin{aligned} \dot{p} &= -H_q(p, q), \\ \dot{q} &= H_p(p, q). \end{aligned}$$

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We assume that H satisfies the following conditions:
(H1) $H \in C^1(R^{2n}, R)$ with $H(0, \dots, 0) = 0$,
(H2) $H > 0$ for $z \neq (0, \dots, 0)$
(H3) $\lim_{z \to (0,\dots,0)} \frac{H_{z_i}(z)}{|z|} = 0$,
(H4) $\lim_{|z| \to \infty} \frac{H_{z_i}(z)}{|z|} = \infty$,
(H5) $z \cdot H_z(z) \geq \mu H(z)$, $\forall z$, $\mu > 2$,
(H6) $|H_{z_1}(z)| + \dots + |H_{z_{2n}}(z)| \leq \gamma(|z_1|^\nu + \dots + |z_{2n}|^\nu)$, $\forall z$, $\gamma > 0$, $\nu > 1$, $i = 1, \dots, 2n$.

Let $E = W^{\frac{1}{2},2}(S^1, R^{2n})$. We are looking for 2π periodic weak solutions for (1.1). We observe that the 2π periodic weak solutions of (1.1) coincide with the critical points of the corresponding functional

$$I : E \to R \in C^1,$$

$$I(u) = \frac{1}{2}\int_0^{2\pi} \dot{z} \cdot Jz dt - \int_0^{2\pi} H(z(t))dt. \tag{1.2}$$

In other words, 2π-periodic weak solution of (1.1) is any $z = (p, q) \in E$ such that

$$\int_0^{2\pi} [(\dot{p} + H_q(t, z(t))) \cdot \psi - (\dot{q} - H_p(t, z(t))) \cdot \phi] dt = 0 \qquad \text{for all } \zeta = (\phi, \psi) \in E,$$

and coincide with the critical points of the corresponding functional

$$I(z) = \int_0^{2\pi} p\dot{q} dt - \int_0^{2\pi} H(z(t))dt = A(z) - \int_0^{2\pi} H(z(t))dt, \tag{1.3}$$

where $A(z) = \frac{1}{2}\int_0^{2\pi} \dot{z} \cdot Jz dt$.

Our main result is the following:

THEOREM 1.1. *Assume that H satisfies the conditions $(H1)$-$(H6)$. Then (1.1) has at least one nonconstant 2π periodic solution. Moreover, if H is of class C^k, this solution is of class $C^k(S^1, R^{2n})$.*

The outline of the proof is the following: In section 2, we investigate the properties of the nnlinear term H and the functional I. In section 3, we recall a variational linking theorem and prove that the functional I satisfies the linking geometry. In section 4, prove Theorem 1.1.

2. Properties of the nonlinear term H and the functional I

Let $E = W^{\frac{1}{2},2}(S^1, R^{2n})$. The scalar product in L^2 extends as the duality pairing between E and $E' = W^{-\frac{1}{2},2}(S^1, R^{2n})$. We know that if $z \in L^2(S^1, R^{2n})$, then it has a Fourier expansion $z(t) = \sum_{k=-\infty}^{k=+\infty} a_k e^{ikt}$, with $a_k \in C^{2n}$, $a_{-k} = \bar{a_k}$ and $\sum_{k\in Z} |a_k|^2 < \infty$. Then E is the closure of such functions with respect to the norm

$$\|z\| = (\sum_{k\in Z}(1+|k|)|a_k|^2)^{\frac{1}{2}}.$$

Let $e_1, \cdots, e_{2n}$ denote the usual bases in R^{2n} and set

$$E^0 = span\{e_1, \cdots, e_{2n}\},$$

$$E^+ = span\{(\sin jt)e_k - (\cos jt)e_{k+n}, (\cos jt)e_k + (\sin jt)e_{k+n}, \\ \mid j \in N, 1 \leq k \leq n\},$$

$$E^- = span\{(\sin jt)e_k + (\cos jt)e_{k+n}, (\cos jt)e_k - (\sin jt)e_{k+n} \\ \mid j \in N, 1 \leq k \leq n\}.$$

Then $E = E^0 \oplus E^+ \oplus E^-$ and E^0, E^+, E^- are the subspaces of E on which A is null, positive definite and negative definite, and these spaces are orthogonal with respective to the bilinear form

$$B[z, \zeta] \equiv \int_0^{2\pi} p \cdot \dot{\psi} + \phi \cdot \dot{q} dt$$

associated with A. Here $z = (p, q)$ and $\zeta = (\phi, \psi)$. If $z \in E^+$ and $\zeta \in E^-$, then the bilinear form is zero and $A(z+\zeta) = A(z) + A(\zeta)$. We also note that E^0, E^+ and E^- are mutually orthogonal in $L^2(S^1, R^{2n})$. Let P^+ be the projection from E onto E^+ and P^- the one from E onto E^-. Then the norm in E is given by

$$\|z\|^2 = |z^0|^2 + A(z^+) - A(z^-) = |z^0|^2 + \|P^+z\|^2 + \|P^-z\|^2$$

which is equivalent to the usual one. The space E with this norm is a Hilbert space.

We need the following facts which is proved in [5]:

Proposition 1. *For each $s \in [1, \infty)$, E is compactly embedded in $L^s(S^1, R^{2n})$. In particular, there is an $\alpha_s > 0$ such that*

$$\|z\|_{L^s} \leq \alpha_s \|z\|$$

for all $z \in E$.

By (H3) and (H5), we obtain the lower bound for $H(z)$ in the term of $|z_1|^\mu + \ldots + |z_{2n}|^\mu$.

Lemma 2.1. *Assume that H satisfies the conditions $(H1) - (H5)$. Then there exist a_0, $b_0 \in R$ with $a_0 > 0$ such that*

$$H(z) \geq a_0(|z|^\mu) - b_0, \qquad \forall z \in E. \tag{2.1}$$

Proof. Let $z \in E$ be such that $|z|^2 \geq R^2$. Let us set $\varphi(\xi) = H(\xi z)$ for $\xi \geq 1$. Then

$$\varphi(\xi)' = z \cdot H_z(\xi z) \geq \frac{\mu}{\xi}\varphi(\xi).$$

Multiplying by $\xi^{-\mu}$, we get

$$(\xi^{-\mu}\varphi(\xi))' \geq 0,$$

hence $\varphi(\xi) \geq \varphi(1)\xi^\mu$ for $\xi \geq 1$. Thus we have

$$H(z) \geq H\big(\frac{R|z|}{\sqrt{|z|^2}}\big)\big(\frac{\sqrt{|z|^2}}{R}\big)^\mu$$

$$\geq c_0\big(\frac{\sqrt{|z|^2}}{R}\big)^\mu \geq a_0(|z|^\mu) - b_0, \text{ for some } a_0, b_0,$$

where $c_0 = \inf\{H(z)|\ |z|^2 = R^2\}$. □

Lemma 2.2. *Assume that H satisfies the conditions $(H1) - (H6)$. Then*
(i) $\int_0^{2\pi} H(0, \ldots, 0)dt = 0$, $\int_0^{2\pi} H(z)dt > 0$ if $z(t) \neq (0, \ldots, 0)$,
$\mathrm{grad}(\int_0^{2\pi} H(z))dt = o(\|z\|)$ as $z \to (0, \ldots, 0)$;
(ii) $z \to \mathrm{grad}(\int_0^{2\pi} H(z(t))dt$ is a compact map;
(iii) if $\int_0^{2\pi} z \cdot H_z(z)dt - 2\int_0^{2\pi} H(z(t))dt = 0$, then
$\mathrm{grad}(\int_0^{2\pi} H(z)dt) = 0$;
(iv) if $\|z_n\| \to +\infty$ and
$\frac{\int_0^{2\pi} z_n \cdot H_z(z_n)dt - 2\int_0^{2\pi} H(z_n)dt}{\|z\|} \to 0$,
then there exists $(z_{h_n})_n$ and $w \in E$ such that

$$\frac{\mathrm{grad}(\int_0^{2\pi} H(z_n)dt)}{\|z_{h_n}\|} \to w \text{ and } \frac{z_{h_n}}{\|z_{h_n}\|} \rightharpoonup (0, \ldots, 0).$$

Proof. (i) (i) follows from (H1), (H2) and (H6), since $1 < \nu$.
(ii) (ii) is easily obtained with standard arguments.
(iii) (iii) is implied by (H5) and the fact that $H(z) > 0$ for $z \neq (0, \ldots, 0)$.
(iv) By Lemma 2.1 and (H5), for $z \in E$,

$$\int_0^{2\pi} z \cdot H_z(z)dt - 2\int_0^{2\pi} H(z)dt \geq$$

$$(\mu - 2)\int_0^{2\pi} H(z)dt \geq (\mu - 2)(a_0\|z\|^{\mu}_{L^{\mu}} - b_1).$$

By (H6),

$$\|\mathrm{grad}(\int_0^{2\pi} H(z)dt)\| \leq C'\|H_z(z)\|_{L^r} \leq C''\||z|^{\nu}\|_{L^r}, \text{ for some } 1 < r < 2$$

and suitable constants C', C''. To get the conclusion it suffices to estimate $\|\frac{|z|^{\nu}}{\|z\|}\|_{L^r}$ in terms of $\frac{\|z\|^{\mu}_{L^{\mu}}}{\|Z\|}$. If $\mu \geq r\nu$, then this is an consequence of Hölder inequality. If $\mu < r\nu$, by the standard interpolation arguments, it follows that $\|\frac{|z|^{\nu}}{\|z\|}\|_{L^r} \leq C\big(\frac{\|z\|^{\mu}_{L^{\mu}}}{\|z\|}\big)^{\frac{\nu}{\mu}}\|z\|^l$, where l is such that $l = -1+\frac{\nu}{\mu}$. Thus we prove (iv). $\square$

Proposition 2. *Assume that $H(z) \in C^1(R^{2n}, R)$. Then $I(z)$ is C^1, that is, $I(z)$ is continuous and Fréchet differentiable in E with Fréchet derivative*

$$\begin{aligned} DI(z)\omega &= \int_0^{2\pi} (\dot{z} - J(H_z(z))) \cdot J\omega \\ &= \int_0^{2\pi} [(\dot{p} + H_q(z)) \cdot \psi - (\dot{q} - H_p(z)) \cdot \phi]dt, \end{aligned}$$

where $z = (p, q)$ and $\omega = (\phi, \psi) \in E$. Moreover the functional $z \mapsto \int_0^{2\pi} H(z(t))dt$ is C^1.

Proof. For $z, w \in E$,

$$
\begin{aligned}
|I(z &+ w) - I(z) - DI(z)w| \\
&= |\frac{1}{2}\int_0^{2\pi} (\dot{z}+\dot{w})\cdot J(z+w) - \int_0^{2\pi} H(z+w) \\
&\quad -\frac{1}{2}\int_0^{2\pi} \dot{z}\cdot Jz + \int_0^{2\pi} H(z) - \int_0^{2\pi} (\dot{z} - J(H_z(z)))\cdot Jw| \\
&= |\frac{1}{2}\int_0^{2\pi} [\dot{z}\cdot Jw + \dot{w}\cdot Jz + \dot{w}\cdot Jw] \\
&\quad - \int_0^{2\pi} [H(z+w) - H(z)] - \int_0^{2\pi} [\dot{z} - J(H_z(z))\cdot Jw]|.
\end{aligned}
$$

We have

$$
|\int_0^{2\pi} [H(z+w) - H(z)]| \le |\int_0^{2\pi} [H_z(z)\cdot w + o(|w|)]dt| = O(|w|).
$$

Thus we have

$$
|I(z+w) - I(z) - DI(z)w| = O(|w|^2).
$$

Next we prove that $I(z)$ is continuous. For $z, w \in E$,

$$
\begin{aligned}
| \;\; I \;\; &(z+w) - I(z)| \\
&= |\frac{1}{2}\int_0^{2\pi} (\dot{z}+\dot{w})\cdot J(z+w) - \int_0^{2\pi} H(z+w) \\
&\quad -\frac{1}{2}\int_0^{2\pi} \dot{z}\cdot Jz + \int_0^{2\pi} H(z)| \\
&= |\frac{1}{2}\int_0^{2\pi} [\dot{z}\cdot Jw + \dot{w}\cdot Jz + \dot{w}\cdot Jw] - \int_0^{2\pi} [H(z+w) - H(z)]| \\
&= O(|w|).
\end{aligned}
$$

Similarly, it is easily checked that I is C^1. □

3. Linking geometry

We recall the variational linking theorem for strongly indefinite functional (cf. [5]) which is a crucial role for proving the existence of the nonconstant 2π periodic weak solution of (1.1).

LEMMA 3.1. *(Variational Linking Theorem)*
Let E be a real Hilbert space with $E = E_1 \oplus E_2$ and $E_2 = E_1^{\perp}$. We suppose that
(I1) $I \in C^1(E, R)$, satisfies $(P.S.)$ condition, and
(I2) $I(u) = \frac{1}{2}(Lu, u) + bu$, where $Lu = L_1P_1u + L_2P_2u$ and $L_i : E_i \to E_i$ is bounded and selfadjoint, $i = 1, 2$,
(I3) b' is compact, and
(I4) there exists a subspace $\tilde{E} \subset E$ and sets $S \subset E, T \subset \tilde{E}$ and constants $\alpha > w$ such that,
(i) $S \subset E_1$ and $I|_S \geq \alpha$,
(ii) T is bounded and $I|_{\partial T} \leq w$,
(iii) S and ∂T link.
Then I possesses a critical value $c \geq \alpha$.

Let $(E_n)_n$ be a sequence of closed subspaces of E with the conditions:

$$E_n = E_n^- \oplus E^0 \oplus E_n^+, \text{ where } E_n^+ \subset E^+,\ E_n^- \subset E^- \text{ for all } n, \tag{3.1}$$

$(E_n^+$ and E_n^- are subspaces of $E), \dim E_n < +\infty, E_n \subset E_{n+1}, \cup_{n \in N} E_n$ is dense in E. Let P_{E_n} be the orthogonal projections from E onto E_n.

Let us prove that the functional I satisfies the linking geometry.

LEMMA 3.2. *Assume that H satisfies the conditions (H1)-(H6). Then*
(i) there exist a small number $\rho > 0$ and a small ball $B_\rho \subset E^0 \oplus E^+$ with radius ρ such that if $U \in \partial B_\rho$, then

$$\alpha = \inf I(z) > 0,$$

(ii) there is an $e \in E^0 \oplus E^+$ and $R > \rho$ such that if

$$W = (\bar{B_R} \cap (E^0 \oplus E^-)) \oplus \{re|\ 0 < r < R\}$$

and $z \in \partial W$, then

$$\sup_{z \in \partial W} I(z) \leq 0$$

and $\beta = \sup_W I(z) < \infty$.

Proof. (i) We note that

$$\text{if } \ z \in E^+, \text{ then } \int_0^{2\pi} \dot{z} \cdot Jzdt \geq \tau_1,$$

$$\text{if } \ z \in E^-, \text{ then } \int_0^{2\pi} \dot{z} \cdot Jzdt \leq -\tau_2$$

for some $\tau_1 > 0$, $\tau_2 > 0$. By (H5) and (H6), $|H(z)| \leq a|z|^b$ for some $a > 0$ and $b > 2$. If $z \in E^0 \oplus E^+$, then we have

$$I(z) = \frac{1}{2}\int_0^{2\pi} \dot{z} \cdot Jzdt - \int_0^{2\pi} H(z)dt$$
$$\geq \tau_1 - a\|z\|_{L^2}^b.$$

Since $b > 2$, there there exist a small number $\rho > 0$ and a small ball B_ρ with radius ρ such that if $z \in \partial B_\rho$, then $\alpha = \inf I(z) > 0$. Thus the assertion (1) hold.

(ii) By Lemma 2.1, there exist a_0, $b_0 \in R$, $a_0 > 0$ such that $H(z) \geq a_0|z|^\mu - b_0$, $\forall z \in E$. Let us choose an element $e \in B_1 \subset E^+$ with $\|e\| = 1$ and $z \neq (0,\dots,0) \in E^0 \oplus E^- \oplus \{re|\ 0 < r\}$. Then $z = w + re$, $w \in E^0 \oplus E^-$, $w \neq (0,\dots,0)$. Then we have

$$I(z) = \frac{1}{2}\int_0^{2\pi} \dot{w}Jw - \frac{1}{2}\int_0^{2\pi} \dot{re}J(re) - \int_0^{2\pi} H(w + re)dt$$
$$\leq -\tau_2 + \frac{1}{2}r^2 - a_0r^\mu - a_0\|w\|^\mu - b_0$$

for some $a_0 > 0$, a_0, $b_0 \in R$. Since $\mu > 2$ and $w \in E^0 \oplus E^-$, there exists $R > 0$ such that if

$$W = (\bar{B_R} \cap (E^0 \oplus E^-)) \oplus \{re|\ 0 < r < R\}$$

and $z \in \partial W$, then $\sup_{z\in\partial W} I(z) < 0$. Moreover $\sup_W I(z) < \frac{1}{2}r^2 < \infty$. Thus the assertion (ii) hold. So the lemma is proved. □

We shall prove that the functional I satisfies the $(P.S.)_c^*$ condition for any $c \in R$.

Lemma 3.3. *Assume that H satisfies the conditions (H1)-(H6). Then the functional I satisfies the $(P.S.)_c^*$ condition with respect to $(E_n)_n$ for any real number c.*

Proof. Let $c \in R$ and (h_n) be a sequence in N such that $h_n \to +\infty$, $(z_n)_n$ be a sequence such that

$$z_n \in E_{h_n}, \forall n, I(z_n) \to c, P_{E_{h_n}}\nabla I(z_n) \to 0.$$

We claim that $(z_n)_n$ is bounded. By contradiction we suppose that $\|z_n\| \to +\infty$ and set $\hat{z_n} = \frac{z_n}{\|z_n\|}$. Then

$$\langle P_{E_{h_n}}\nabla I(z_n), \hat{z_n}\rangle = \langle\nabla I(z_n), \hat{z_n}\rangle = 2\frac{I(z_n)}{\|z_n\|} -$$

$$\frac{\int_0^{2\pi} H_z(z_n) \cdot z_n dt - 2\int_0^{2\pi} H(z)dt}{\|z_n\|} \longrightarrow 0.$$

Hence

$$\frac{\int_0^{2\pi} H_z(z_n) \cdot z_n dt - 2\int_0^{2\pi} H(z_n)dt}{\|z_n\|} \longrightarrow 0.$$

By Lemma 2.2,

$$\frac{\text{grad} \int_0^{2\pi} H(z)dt}{\|z_n\|} \qquad \text{converges}$$

and $\hat{z_n} \rightharpoonup 0$. We get

$$\frac{P_{E_{h_n}} \nabla I(z_n)}{\|z_n\|} = P_{E_{h_n}} \hat{z_n} - \frac{P_{E_{h_n}} \text{grad}(\int_0^{2\pi} H(z)dt)}{\|z_n\|} \longrightarrow 0,$$

so $(P_{E_{h_n}} \hat{z_n}$ converges. Since $(\hat{z_n})_n$ is bounded and $(\frac{d}{dt})^{-1}$ is a compact mapping, up to subsequence, $(\hat{z_n})_n$ has a limit. Since $\hat{z_n} \rightharpoonup (0, \ldots, 0)$, we get $\hat{z_n} \rightarrow (0, \ldots, 0)$, which is a contradiction to the fact that $\|\hat{z_n}\| = 1$. Thus $(z_n)_n$ is bounded. We can now suppose that $z_n \rightharpoonup z$ for some $z \in E$. Since the mapping $z \mapsto \text{grad}(\int_0^{2\pi} H(z)dt)$ is a compact mapping, $\text{grad}(\int_0^{2\pi} H(z_n)dt) \longrightarrow \text{grad}(\int_0^{2\pi} H(z)dt)$. Thus $(P_{E_{h_n}} \dot{z_n})$ converges. Since $(\frac{d}{dt})^{-1}$ is a compact operator and $(z_n)_n$ is bounded, we deduce that, up to a subsequence, $(z_n)_n$ converges to some z strongly with $\nabla I(z) = \lim \nabla I(z_n) = 0$. Thus we prove the lemma. $\square$

4. Proof of theorem 1.1

Assume that H satisfies the conditions (H1)-(H6). We note that $I(0, 0) = 0$. By Proposition 2.2, $I(z) \in C^1$. By (ii) of Lemma 2.2, $z \mapsto \text{grad}(\int_0^{2\pi} H(z)dt$ is a compact mapping. By Lemma 3.2, there exist a small number $\rho > 0$ and a small ball $B_\rho \subset E^0 \oplus E^+$ with radius ρ such that if $z \in \partial B_\rho$, then $\alpha = \inf I(z) > 0$, and there is an $e \in E^0 \oplus E^+$ and $R > \rho$ such that if

$$W = (\bar{B}_R \cap (E^0 \oplus E^-)) \oplus \{re| \ 0 < r < R\},$$

then

$$\sup_{z \in \partial W} I(z) \leq 0.$$

Let us set $\beta = \sup_W I$. We note that $\beta < +\infty$. Let $(E_n)_n$ be a sequence of subspaces of E satisfying (3.2). Clearly $E^0 \subset E_n$ for all n, and ∂B_ρ and ∂W link. We have, for all $n \in N$,

$$\sup_{\partial W \cap E_n} I < \inf_{\partial B_\rho \cap E_n} I.$$

Moreover, by Lemma 3.3, $I_n = I|_{E_n}$ satisfies the $(P.S.)_c^*$ condition for any $c \in R$. Thus by Lemma 3.1 (Variational Linking Theorem), there exists a critical point z_n for I_n with

$$\alpha \leq \inf_{\partial B_\rho \cap E_n} I \leq I(z_n) \leq \sup_{W \cap E_n} I \leq \beta.$$

Since I_n satisfies the $(P.S.)_c^*$ condition, we obtain that, up to a subsequence, $z_n \to z$, with z a critical point for I such that $\alpha \leq I(z) \leq \beta$. Hence $z \neq (0,0)$. Thus system (1.1) has a nontrivial solution. Thus Theorem 1.1 is proved.

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