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MOUNTAIN PASS GEOMETRY APPLIED TO THE NONLINEAR MIXED TYPE ELLIPTIC PROBLEM

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Abstract. We show the existence of at least one nontrivial solution of the homogeneous mixed type nonlinear elliptic problem. Here mixed type nonlinearity means that the nonlinear part contain the jumping nonlinearity and the critical growth nonlinearity. We first investigate the sub-level sets of the corresponding functional in the Soboles space and the linking inequalities of the functional on the sub-level sets. We next investigate that the functional I satisfies the mountain pass geometry in the critical point theory. We obtain the result by the mountain pass method, the critical point theory and variational method.

1. Introduction

In this paper we investigate the multiple solutions of the following elliptic problem with jumping and critical growth nonlinearity

$$\Delta u + bu^{+} + p|u|^{p-1} = 0 \qquad \text{in } \ \Omega, \tag{1.1}$$
$$u = 0 \qquad \text{on } \partial\Omega,$$

where Ω is a bounded subset of R^n with smooth boundary, $2 < p < 2^*$, $2^* = \frac{2n}{n-2}$, $n \geq 3$, $u^+ = \max\{u, 0\}$, $u^- = -\min\{u, 0\}$, $u(x) \in W_0^{1,2}(\Omega)$.

This mixed type nonlinear problem contains the jumping nonlinearity and the critical growth nonlinearity. The authors [1], [2], [4], [5], [9], [10], [11] consider the jumping nonlinear problem. They investigate the multiplicity results when the constant b of the nonlinear term is less than λ_1 or lies in the between λ_k and λ_{k+1}, $k \geq 1$. They obtain the

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multiplicity results by use of the Leray-Schauder degree theory, geometry of the mapping defined on the finite dimensional reduction subspace, mountain pass geometry in the critical point theory, the category theory in critical point theory. In [3], [6], [7], [8], [11] the authors also considered the critical growth nonlinear problem. They consider the multiplicity results by use of the variational method, the critical point theory and the category theory in the critical point theory. In this paper the authors consider the mixed type case and investigate the multiplicity results when the jumping nonlinearity and the critical growth nonlinearity act on the equation.

The eigenvalue problem

$$-\Delta u = \lambda u, \qquad \text{in } \Omega, \tag{1.2}$$

$$u = 0 \qquad \text{on } \partial\Omega$$

has infinitely many eigenvalues λ_k, $k \geq 1$ with $\lambda_1 < \lambda_2 \leq \ldots \leq \lambda_k \leq \ldots$ and infinitely many eigenfunction ϕ_k belonging to the eigenvalue λ_k, $k \geq 1$. Let H be a Sobolev space $W_0^{1,2}(\Omega)$ with the norm

$$\|u\|^2 = \int_\Omega |\nabla u(x)|^2 dx.$$

In this paper we are looking for the weak solutions of (1.1) in H, that is, $u \in H$ such that

$$\int_\Omega (\Delta u + bu^+) v dx + p \int_\Omega |u|^{p-1} v dx = 0 \quad \text{for all} \quad v \in H.$$

Our main result is the following:

THEOREM 1.1. *Assume that* $\lambda_1 < b < \lambda_2$ *and* $2 < p < 2^*$, $2^* = \frac{2n}{n-2}$, $n \geq 3$. *Then (1.1) has at least one nontrivial solution.*

In section 2 we obtain some results for the Sobolev norm and the operator $-\Delta$. We also obtain the result that the corresponding functional $I(u)$ belongs to C^1. In section 3 we investigate the sub-level sets of the functional and the linking inequalities of the functional on the sub-level sets. We also investigate that the functional $I(u)$ satisfies the mountain pass geometry. We prove the main result by the mountain pass method in the critical point theory.

2. Some results on the operator $-\Delta$ and the functional I

LEMMA 2.1. *Let $u \in H = W_0^{1,2}(\Omega, R)$ and $\|\cdot\|$ be a Sobolev norm. Then*
(i) $\|u\| \geq C\|u\|_{L^2(\Omega)}$ for some constant $C > 0$.
(ii) $\|u\| = 0$ if and only if $\|u\|_{L^2(\Omega)} = 0$.
(iii) $-\Delta u \in H$ implies $u \in H$.
(iv) Let c be not an eigenvalue of $-\Delta$ and $f \in H$. Then all the solutions of

$$(-\Delta - c)u = f$$

belong to H.

Proof. (i) and (ii) can be checked easily by the definition of $\|\cdot\|$.
(iii) Let $-\Delta u = f \in W_0^{1,2}(\Omega, R)$. Then f is of the form $f = \sum h_m\phi_m$. Then

$$(-\Delta)^{-1} f = \sum \frac{1}{\lambda_m} h_m \phi_m.$$

We note that for any c, $\{\lambda_m : \ \lambda_m < |c|\}$ is finite. Thus we have

$$\|(-\Delta)^{-1} f\|^2 = \sum \lambda_m^2 \frac{1}{\lambda_m^2} h_m^2 \leq \sum h_m^2,$$

which means that

$$\|(-\Delta)^{-1} f\| \leq \|f\|_{L^2(\Omega)}.$$

(iv) (iv) comes from (iii). □

LEMMA 2.2. *Assume that $\lambda_1 < b$ and b is not an eigenvalue of $-\Delta$ with Dirichlet boundary condition. Then*

$$\Delta u + bu^+ = 0 \ \ \textit{in} \ \ H \tag{2.1}$$

has only the trivial solution $u = 0$.

Proof. We note that $u = 0$ is a solution of (2.1). We rewrite (2.1) as

$$(-\Delta - \lambda_1)u = (b - \lambda_1)u^+ + \lambda_1 u^- \ \ \text{in} \ \ H.$$

We note that $((-\Delta - \lambda_1)u, \phi_1) = 0$. Thus we have

$$\int_\Omega [(b - \lambda_1)u^+ + \lambda_1 u^-]\phi_1 dx = 0. \tag{2.2}$$

Since $\lambda_1 < b$, $(b-\lambda_1)u^+ + \lambda_1 u^-$ is greater than or equal to 0 and strictly greater than zero if u is strictly greater than zero. The only possibility to hold (2.2) is that $u = 0$. That is, $u = 0$ is the only solution of (2.1). □

By the following Proposition 2.1, the weak solutions of (1.1) coincide with the critical points of the corresponding functional

$$I \in C^1(H, R),$$

$$I(u) = \int_\Omega [\frac{1}{2}|\nabla u|^2 - \frac{b}{2}|u^+|^2 - |u|^p]dx. \tag{2.3}$$

PROPOSITION 1. *Assume that $\lambda_1 < b$, b is not an eigenvalue. Then the functional $I(u)$ is continuous, $Fr\acute{e}chet$ differentiable in H with $Fr\acute{e}chet$ derivative*

$$\nabla I(u)v = \int_\Omega [(-\Delta u)\cdot v - bu^+ \cdot v - p|u|^{p-1}\cdot v]dx.$$

Moreover $\nabla I \in C$. That is, $I \in C^1$.

Proof. First we will prove that $I(u)$ is continuous at u. For u, $v \in H$,

$$\begin{aligned}
|I(u+v) - I(u)| &= |\frac{1}{2}\int_\Omega (-\Delta u - \Delta v)\cdot(u+v)dx \\
&\quad - \int_\Omega [\frac{b}{2}|(u+v)^+|^2 + |u+v|^p]dx \\
&\quad -\frac{1}{2}\int_\Omega (-\Delta u)\cdot u dx + \int_\Omega [\frac{b}{2}|u^+|^2 + |u|^p]dx| \\
&= |\frac{1}{2}\int_\Omega (-\Delta u \cdot v - \Delta v \cdot u - \Delta v \cdot v)dx \\
&\quad - \int_\Omega (\frac{b}{2}|(u+v)^+|^2 + |u+v|^p \\
&\quad -\frac{b}{2}|u^+|^2 - |u|^p)dx|.
\end{aligned}$$

Let $u = \sum h_n\phi_n$, $v = \sum k_n\phi_n$. Then we have

$$|\int_\Omega (-\Delta u)\cdot v dx| = |\sum \lambda_n h_n k_n| \le \|u\|\cdot\|v\|,$$

$$|\int_\Omega (-\Delta v)\cdot u dx| = |\sum \lambda_n k_n h_n| \le \|u\|\cdot\|v\|,$$

$$|\int_\Omega (-\Delta v)\cdot v dx| = |\sum \lambda_n k_n k_n| \le \|v\|^2,$$

from which we have

$$|\frac{1}{2}\int_{\Omega}(-\Delta u\cdot v-\Delta v\cdot u-\Delta v\cdot v)dx|\leq\|u\|\cdot\|v\|+\|v\|^{2}. \tag{2.4}$$

On the other hand

$$||(u+v)^{+}|^{2}-|u^{+}|^{2}|\leq 2u^{+}|v|+|v|^{2},$$
$$||u+v|^{p}-|u|^{p}|\leq C_{1}|u|^{p-1}||v|+R_{2}(|u|,|v|),$$

where $R_2(|u|,|v|)$ is the remainder part of the Taylor's expansion series. Hence we have

$$\begin{aligned}|\int_{\Omega}(|(u+v)^{+}|^{2}-|u^{+}|^{2})dx| &\leq 2\|u^{+}\|_{L^{2}(\Omega)}\|v\|_{L^{2}(\Omega)}+\|v\|^{2}_{L^{2}(\Omega)}\\ &\leq 2\|u\|\cdot\|v\|+\|v\|^{2},\end{aligned} \tag{2.5}$$

(2.6)

$$\begin{aligned}|\int_{\Omega}(|u+v|^{p}-|u|^{p})dx| &\leq C_{1}\|u\|^{p-1}_{L^{2}(\Omega)}\|v\|_{L^{2}(\Omega)}+R_{2}(\|u\|_{L^{2}(\Omega)},\|v\|_{L^{2}(\Omega)})\\ &\leq C_{2}\|u\|^{p-1}\|v\|+R_{2}(\|u\|,\|v\|).\end{aligned}$$

Combining (2.4) with (2.5) and (2.6), we have

$$|I(u+v)-I(u)|=o(\|v\|^{2})$$

from which we can conclude that $I(u)$ is continuous at u. Next we shall prove that $I(u)$ is *Fréchet* differentiable in H. For u, $v\in H$,

$$\begin{aligned}&|I(u+v)-I(u)-\nabla I(u)v|\\ =\ &|\frac{1}{2}\int_{\Omega}(-\Delta u-\Delta v)\cdot(u+v)dx-\int_{\Omega}[\frac{b}{2}|(u+v)^{+}|^{2}+|u+v|^{p}]dx\\ &-\frac{1}{2}\int_{\Omega}(-\Delta u)\cdot udx+\int_{\Omega}[\frac{b}{2}|u^{+}|^{2}+|u|^{p}]dx\\ &-\int_{\Omega}(-\Delta u-bu^{+}-p|u|^{p-1})\cdot vdx|\\ =\ &|\int_{\Omega}[\frac{1}{2}(-\Delta v)\cdot v-\frac{b}{2}|(u+v)^{+}|^{2}-|u+v|^{p}\\ &+\frac{b}{2}|u^{+}|^{2}+|u|^{p}+bu^{+}v+p|u|^{p-1}v]dx|.\end{aligned}$$

Combining (2.4) with (2.5) and (2.6), we have that

$$|I(u+v)-I(u)-\nabla I(u)v|=O(\|v\|^{2}). \tag{2.7}$$

Thus $I(u)$ is *Fréchet* differentiable in H. Similarly, it is easily checked that $I \in C^1$. $\square$

3. Proof of Theorem 1.1

Now we shall show that the functional I satisfies the mountain pass geometry. Let us set

$$X = \text{span}\{\phi_1\}, \quad Y = X^{\perp}.$$

Then X is one dimensional subspace and

$$H = X \oplus Y.$$

We have the following linking inequalities:

LEMMA 3.1. *Assume that $\lambda_1 < b < \lambda_2$. Then there exist $\rho > 0$ and a small ball B_ρ with radius ρ such that $B_\rho \cap Y \neq \emptyset$,*

$$\inf_{u \in \partial B_\rho \cap Y} I(u) > 0 \quad \textit{and} \quad \inf_{u \in B_\rho \cap Y} I(u) > -\infty.$$

Proof. Let $u \in Y$. Then we have

$$\begin{aligned} I(u) &= \int_\Omega [\frac{1}{2}|\nabla u|^2 - \frac{b}{2}|u^+|^2 - |u|^p]dx \\ &\geq \int_\Omega [\frac{1}{2}|\nabla u|^2 - \frac{b}{2}|u|^2 - |u|^p]dx \\ &\geq \frac{1 - \frac{b}{\lambda_2}}{2}\|u\|^2 - \int_\Omega |u|^p dx. \end{aligned}$$

Let us define

$$C_p(\Omega) = \inf_{u \in H \setminus \{0\}} \frac{\int_\Omega |\nabla u|^2 dx}{(\int_\Omega |u|^p dx)^{\frac{2}{p}}}.$$

Then we have

$$I(u) \geq \frac{1 - \frac{b}{\lambda_2}}{2}\|u\|^2 - (C_p(\Omega))^{-\frac{p}{2}}\|u\|^p.$$

Since $\lambda_2 - b > 0$ and $p > 2$, there exist a small number $\rho > 0$ and a small ball B_ρ with radius ρ such that $\inf_{u \in \partial B_\rho \cap Y} I(u) > 0$ and $\inf_{u \in B_\rho \cap Y} I(u) > -(C_p(\Omega))^{-\frac{p}{2}}\|u\|^p > -\infty$. $\square$

LEMMA 3.2. *Assume that* $\lambda_1 < b < \lambda_2$. *Then we can choose* $e \in \partial B_1 \cap Y$, $R > 0$ *and* $Q \equiv (\bar{B_R} \cap X) \oplus \{\sigma e | \ 0 < \sigma < R\}$ *such that*

$$\sup_{u \in \partial Q} I(u) < 0 \quad and \quad \sup_{u \in Q} I(u) < \infty.$$

Proof. Let $u \in X \oplus \{\sigma e | \ \sigma > 0\}$, $u = v + \sigma e$, $v \in X$, $e \in B_1 \cap Y$. We note that

$$\text{if } \ u \in X, \text{ then } \int_\Omega [|\nabla u^+|^2 - \frac{b}{2}|u^+|^2]dx \leq \frac{1 - \frac{b}{\lambda_1}}{2}\|u^+\|^2 < 0.$$

For $s > 0$ we have

$$\begin{aligned} I(su) &= s^2(\int_\Omega [\frac{1}{2}|\nabla(v + \sigma e)|^2 - \frac{b}{2}|(v + \sigma e)^+|^2]dx - s^p \int_\Omega |v + \sigma e|^p dx \\ &\leq \frac{s^2(1 - \frac{b}{\lambda_1})}{2}\|v^+\|^2 + \frac{s^2(1 - \frac{b}{\lambda_n})}{2}\sigma^2 - s^p \int_\Omega |v + \sigma e|^p]dx \end{aligned}$$

for some $\lambda_n \geq \lambda_2$. Since $p > 2$, $I(su) = I(s(v + \sigma e)) \to -\infty$ as $s \to \infty$. Thus there exist $R > 0$, a ball B_R and $Q \equiv (\bar{B_R} \cap X) \oplus \{\sigma e | \ 0 < \sigma < R\}$ such that if $u \in \partial Q$, then $\sup I(u) < 0$. Moreover if $u \in Q$ then $\sup I(u) < \frac{s^2(1 - \frac{b}{\lambda_n})}{2}\sigma^2 < \infty$. Thus we prove the lemma. □

LEMMA 3.3. *Assume that* $\lambda_1 < b < \lambda_2$. *Then* I *satisfies the* $(P.S.)_c$ *condition for every real number* $c \in R$.

Proof. Let $c \in R$ and $(u_n)_n$ be a sequence such that

$$u_n \in H, \ \forall n, \ I(u_n) \to c, \ \nabla I(u_n) \to 0.$$

We claim that $(u_n)_n$ is bounded. By contradiction we suppose that $\|u_n\| \to +\infty$ and set $\hat{u_n} = \frac{u_n}{\|u_n\|}$. Then we have

$$\begin{aligned} &\langle \nabla I(u_n), \hat{u_n} \rangle \\ &= \frac{2I(u_n)}{\|u_n\|} - \frac{\int_\Omega [bu_n^+ + p|u_n|^{p-1}] \cdot u_n dx - 2\int_\Omega [\frac{b}{2}|u_n^+|^2 + |u_n|^p]dx}{\|u_n\|} \longrightarrow 0. \end{aligned}$$

Hence

$$\frac{\int_\Omega [bu_n^+ + p|u_n|^{p-1}] \cdot u_n dx - 2\int_\Omega [\frac{b}{2}|u_n^+|^2 + |u_n|^p]dx}{\|u_n\|} \longrightarrow 0.$$

We note that

$$\int_\Omega [bu_n^+ + p|u_n|^{p-1}] \cdot u_n dx - 2\int_\Omega [\frac{b}{2}|u_n^+|^2 + |u_n|^p]dx = (p-2)\int_\Omega |u_n|^p dx,$$

so we have

$$(3.1)\qquad (p-2)\frac{\int_\Omega |u_n|^p dx}{\|u_n\|} = (p-2)\frac{\|u_n\|^p_{L^p(\Omega)}}{\|u_n\|} \longrightarrow 0.$$

Since $p > 2$,

$$(3.2)\qquad \frac{\|u_n\|^p_{L^p(\Omega)}}{\|u_n\|} \to 0.$$

From (3.1), $\hat{u_n} \rightharpoonup 0$. On the other hand

$$\|bu_n^+ + p|u_n|^{p-1}\| \le C_1(\|u_n\| + \||u_n|^{p-1}\|_{L^{2^{*'}}(\Omega)})$$

for suitable constant C_1. Thus we have

$$\|\frac{bu_n^+ + p|u_n|^{p-1}}{\|u_n\|}\| \le C_1(1 + \|\frac{|u_n|^{p-1}}{\|u_n\|}\|_{L^{2^{*'}}(\Omega)}).$$

If $p \ge 2^{*'}(p-1)$, then by the $H\ddot{o}lder's$ inequality, it is easily checked that $\|\frac{|u_n|^{p-1}}{\|u_n\|}\|_{L^{2^{*'}}(\Omega)}$ can be estimated in terms of $\frac{\|u_n\|^p_{L^p(\Omega)}}{\|u_n\|}$. If $p \le 2^{*'}(p-1)$, then by the standard interpolation inequalities, $\|\frac{|u_n|^{p-1}}{\|u_n\|}\|_{L^{2^{*'}}(\Omega)} \le C_2(\frac{\|u_n\|^p_{L^p(\Omega)}}{\|u_n\|})^{\frac{(p-1)\alpha}{p}}\|u_n\|^\beta$ for some constant C_2, where $\alpha > 0$ is such that $\frac{\alpha}{p} + \frac{1-\alpha}{2^*} = \frac{1}{2^{*'}}$ and $\beta = (1-\alpha)(p-1) - 1 - \frac{(p-1)\alpha}{p}$. Since $p - 1 \le 2^* - 1 - (2^* - p)(1 - \frac{2^{*'}}{2^*})$, $\beta < 0$. Thus we have

$$\|\frac{bu_n^+ + p|u_n|^{p-1}}{\|u_n\|}\| \le C_2(1 + (\frac{\|u_n\|^p_{L^p(\Omega)}}{\|u_n\|})^{\frac{(p-1)\alpha}{p}}\|u_n\|^\beta).$$

for a constant C_2. By (3.2) and $\beta < 0$,

$$(3.3)\qquad \frac{bu_n^+ + p|u_n|^{p-1}}{\|u_n\|} \text{ converges.}$$

We get

$$\frac{\nabla I(u_n)}{\|u_n\|} = -\Delta\hat{u_n} - \frac{bu_n^+ + p|u_n|^{p-1}}{\|u_n\|} \longrightarrow 0.$$

By (3.3), $-\Delta\hat{u_n}$ converges. Since $(\hat{u_n})_n$ is bounded and the inverse operator of $-\Delta$ is a compact mapping, up to subsequence, $(\hat{u_n})_n$ has a limit. Since $\hat{u_n} \rightharpoonup 0$, we get $\hat{u_n} \to 0$, which is a contradiction to the fact that

$\|\hat{u_n}\| = 1$. Thus $(u_n)_n$ is bounded. We can now suppose that $u_n \rightharpoonup u$ for some $u \in H$. We claim that $u_n \to u$ strongly. We have that

$$\langle \nabla I(u_n), u_n \rangle = (\|u_n\|^2 - \int_\Omega [bu_n^+ u_n + p|u_n|^{p-1} u_n] dx) \longrightarrow 0.$$

Since $u_n \rightharpoonup u$ for some $u \in H$, $\int_\Omega [bu_n^+ u_n + p|u_n|^{p-1} u_n] dx$ converges to $\int_\Omega [bu^+ u + p|u|^{p-1} u] dx$. So $\|u_n\|^2$ converge. Thus $(u_n)_n$ converges to some u strongly with $\nabla I(u) = \lim \nabla I(u_n) = 0$. Thus we prove the lemma. $\square$

Proof of Theorem 1.1

By Proposition 2.1, the functional I belong to $C^1(H, R^1)$. By Lemma 3.1 and Lemma 3.2, there exist $\rho > 0$, a small ball B_ρ with radius ρ, $e \in \partial B_1 \cap Y$ and $Q \equiv (\bar{B_R} \cap X) \oplus \{\sigma e |\ 0 < \sigma < R\}$ such that $B_{rho} \cap Y \neq \emptyset$,

$$\sup_{u \in \partial Q} I(u) < \inf_{u \in \partial B_\rho \cap Y} I(u)$$

and

$$\sup_{u \in Q} I(u) < \infty \quad \text{and} \quad -\infty < \inf_{u \in B_\rho \cap Y} I(u).$$

By Lemma 3.3, the functional $I(u)$ satisfies the $(P.S.)_c$ condition for any $c \in R$. Thus by the Mountain Pass Theorem, I possesses a critical value $c \geq 0$ such that

$$c = \inf_{\gamma \in \Gamma} \max_{u \in Q} I(\gamma(u)),$$

where

$$\Gamma = \{\gamma \in C(\bar{Q}, H) |\ \gamma = id \text{ on } \partial Q\}.$$

Therefore (1.1) has at least one nontrivial solution.

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