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THE CLASS GROUP OF D^*/U FOR D AN INTEGRAL DOMAIN AND U A GROUP OF UNITS OF D

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Abstract. Let D be an integral domain, and let U be a group of units of D. Let $D^* = D - \{0\}$ and $\Gamma = D^*/U$ be the commutative cancellative semigroup under $aU + bU = abU$. We prove that $Cl(D) = Cl(\Gamma)$ and that D is a PvMD (resp., GCD-domain, Mori domain, Krull domain, factorial domain) if and only if Γ is a PvMS (resp., GCD-semigroup, Mori semigroup, Krull semigroup, factorial semigroup). Let $U = U(D)$ be the group of units of D. We also show that if D is integrally closed, then $D[\Gamma]$, the semigroup ring of Γ over D, is an integrally closed domain with $Cl(D[\Gamma]) = Cl(D) \oplus Cl(D)$; hence D is a PvMD (resp., GCD-domain, Krull domain, factorial domain) if and only if $D[\Gamma]$ is.

1. Introduction

Let D be an integral domain with quotient field K, and $U(D)$ be the group of units of D. Let $D^* = D - \{0\}$, $K^* = K - \{0\}$, and U be a subgroup of $U(D)$. The $U(D)$ is a subgroup of the multiplicative group K^*, and the group operation on the factor group $G(D) = K^*/U(D)$ is written as addition $aU(D) + bU(D) = abU(D)$. For $xU(D), yU(D) \in G(D)$, define $xU(D) \leq yU(D)$ if and only if $\frac{y}{x} \in D$. Then the realtion $\leq$ is a partial order on $G(D)$ compatible with its group operation. The group $G(D)$, partially ordered under $\leq$, is called the *group of divisibility of* D. It is well known that $G(D)$ is lattice ordered (resp., totally ordered) if and only if D is a GCD-domain (resp., valuation domain) [3, Theorems 16.2 and 16.3]. It is clear that $D^*/U(D)$ is a semigroup with quotient

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group $G(D)$. More generally, D^*/U is a commutative cancellative semigroup (under addition $aU + bU = abU$) with quotient field K^*/U (see Lemma 1). In this paper, we study the multiplicative t-ideal structures of the semigroup D^*/U via those of the integral domain D.

Let $\Gamma = D^*/U$. For a nonzero fractional ideal I of D and a fractional ideal J of Γ, let $I_s = \{aU | 0 \neq a \in I\}$ and $J_r = (\{x | xU \in J\})$. In this paper, we show that $(I_s)_t = (I_t)_s$; I is a (prime) t-ideal if and only if I_s is a (prime) t-ideal; if I is a t-ideal, then $(I_s)_r = I$; and I is t-invertible if and only if I_s is t-invertible. We also show that $(J_r)_t = (J_t)_r$; J is a (prime) t-ideal if and only if J_r is a (prime) t-ideal; if J is a t-ideal, then $(J_r)_s = I$; and J is t-invertible if and only if J_r is t-invertible. As a corollary, we have that D is a PvMD (resp., GCD-domain, Mori domain, Krull domain, factorial domain) if and only if Γ is a PvMS (resp., GCD-semigroup, Mori semigroup, Krull semigroup, factorial semigroup). Also, we prove that $Cl(D) = Cl(D^*/U)$, i.e., the map $\varphi : Cl(D) \to Cl(D^*/U)$, given by $cl(I) \to cl(I_s)$, is a group isomorphism. We show that D is a PvMD (resp., GCD-domain, Krull domain, factorial domain) if and only if $D[D^*/U(D)]$ is a PvMD (resp., GCD-domain, Krull domain, factorial domain).

We first review some definitions and notations. Let $\mathbf{F}(D)$ be the set of nonzero fractional ideals of D. For each $I \in \mathbf{F}(D)$, let $I^{-1} = \{x \in K | xI \subseteq D\}$, $I_v = (I^{-1})^{-1}$, and $I_t = \cup\{J_v | J$ is a nonzero finitely generated subideal of $I\}$. An $I \in \mathbf{F}(D)$ is called a *v-ideal* (resp., *t-ideal*) if $I_v = I$ (resp., $I_t = I$), while a t-ideal is a maximal t-ideal if it is maximal among proper integral t-ideals of D. It is well known that a prime ideal minimal over a t-ideal is a t-ideal; hence if D is not a field, then t-Spec$(D) \neq \emptyset$, where t-Spec(D) is the set of prime t-ideals of D. An $I \in \mathbf{F}(D)$ is said to be *t-invertible* if $(II^{-1})_t = D$; equivalently, $II^{-1} \nsubseteq P$ for all maximal t-ideals P of D. We say that D is a *Mori domain* if D satisfies the ascending chain condition on integral v-ideals; equivalently, each v-ideal I of D is of finite type, i.e., $I = (a_i, \dots, a_n)_v$ for some $a_i \in D$. It is well known that Krull domains are Mori. The ring D is called a *Prüfer v-multiplication domain* (PvMD) if each nonzero finitely generated ideal of D is t-invertible. The (t-)*class group* of D is an ableian group $Cl(D) = T(D)/Prin(D)$, where $T(D)$ is the group of t-invertible fractional t-ideals of D under the t-multiplication $I * J = (IJ)_t$

and $Prin(D)$ is the subgroup of $T(D)$ of principal fractional ideals. We denote by $cl(I)$ the class of $Cl(D)$ containing I.

Let Γ be a commutative cancellative semigroup. As in the domain case, we can define the v- and t-operation; (maximal, prime) t-ideals; t-Spec(Γ); Mori semigroup; t-invertibility; Prüfer v-multiplication semigroup (PvMS); and the (t-)class group for Γ. The reader can refer to [3, §32 and §34] for the v- and t-operation on integral domains; to [4, §16] or [5, §11] for the v- and t-operation on semigroups; and to [5] for semigroups.

2. $Cl(D) = Cl(D^*/U)$ for U a group of units of D

Throughout D is an integral domain with quotient field K, $D^* = D - \{0\}$, $K^* = K - \{0\}$ and U is a group of units of D (hence U is a subgroup of the multiplicative group K^*).

LEMMA 1. *Let U be a group of units of D, $\Gamma = D^*/U$ and $G = K^*/U$.*
(1) Γ is a commutative cancellative semigroup under addition $aU + bU = abU$.
(2) G is the quotient group of Γ.
(3) Γ is torsion-free if and only if $x^n \in U$ implies $x \in U$ for any $x \in K^*$ and an integer $n \geq 1$.

Proof. This is an easy exercise. □

DEFINITION 2. *Let U be a group of units of D, and $\Gamma = D^*/U$ be the additive semigroup with quotient group $G = K^*/U$. Let I be a nonzero fractional ideal of D and J be a fractional ideal of Γ. Define*

$$I_s = \{aU | 0 \neq a \in I\} \text{ and } J_r = (\{x | xU \in J\}).$$

Clearly, I_s and J_r are fractional ideals of Γ and D, respectively.

For $\{a_\alpha\} \subseteq K^*$, we denote by $(\{a_\alpha\})$ (resp., $[\{a_\alpha U\}]$) the fractional ideal of D (resp., Γ) generated by $\{a_\alpha\}$ (resp., $\{a_\alpha U\}$); hence $(\{a_\alpha\}) = \{\sum a_{\alpha_i} d_i | a_{\alpha_i} \in \{a_\alpha\}$ and $d_i \in D\}$ and $[\{a_\alpha U\}] = \cup_\alpha (a_\alpha U + \Gamma)$. We first study the fractional ideal I_s of D^*/U for a nonzero fractional ideal I of D.

Proposition 3. *Let U be a group of units of D, and let $\Gamma = D^*/U$ be the semigroup. Let I be a nonzero fractional ideal of D and $\{a_\alpha\}$, $\{b_\beta\}$ be nonempty subsets of K^*.*
(1) If $(\{a_\alpha\})_v \subseteq (\{b_\beta\})_v$, then $[\{a_\alpha U\}]_v \subseteq [\{b_\beta U\}]_v$.
(2) $(I_s)^{-1} = (I^{-1})_s$; hence $(I_s)_v = (I_v)_s$.
(3) $(I_s)_t = (I_t)_s$.
(4) I is a t-ideal if and only if I_s is a t-ideal.
(5) $(I_s)_r = I$, and I is a prime ideal if and only if I_s is a prime ideal.
(6) $((I_1I_2)_s)_t = ((I_1)_s + (I_2)_s)_t$ for any $I_1, I_2 \in \mathbf{F}(D)$.
(7) I is a t-invertible t-ideal if and only if I_s is a t-invertible t-ideal.

Proof. (1) Let $x \in K^*$. Then $xU \in [\{b_\beta U\}]^{-1} \Rightarrow xb_\beta U = xU + b_\beta U \in \Gamma$ for all $b_\beta \Rightarrow xb_\beta \in D$ for all $b_\beta \Rightarrow x \in (\{b_\beta\})^{-1} \subseteq (\{a_\alpha\})^{-1}$ by assumption $\Rightarrow xa_\alpha \in D$ for all $a_\alpha \Rightarrow xU + a_\alpha U = xa_\alpha U \in \Gamma$ for all $a_\alpha \Rightarrow xU \in [\{a_\alpha U\}]^{-1}$. Hence $[\{b_\beta U\}]^{-1} \subseteq [\{a_\alpha U\}]^{-1}$, and thus $[\{a_\alpha U\}]_v \subseteq [\{b_\beta U\}]_v$.

(2) Let $y \in K^*$. Then $yU \in (I_s)^{-1} \Leftrightarrow yaU = yU + aU \in \Gamma$ for all $0 \neq a \in I$, $\Leftrightarrow ya \in D$ for all $0 \neq a \in I$, $\Leftrightarrow y \in I^{-1}$, $\Leftrightarrow yU \in (I^{-1})_s$.

(3) Let $y \in K^*$. Then $yU \in (I_s)_t \Leftrightarrow yU \in [\{a_iU\}]_v = ((\{a_i\})_v)_s$ for some finite set $\{a_iU\} \subseteq I_s$ (see (2) for the equality), $\Leftrightarrow y \in (\{a_i\})_v$ for some finite set $\{a_i\} \subseteq I$, $\Leftrightarrow y \in I_t$, $\Leftrightarrow yU \in (I_t)_s$.

(4) If I is a t-ideal, then $(I_s)_t = (I_t)_s = I_s$ by (3). Conversely, assume that I_s is a t-ideal; so $I_s = (I_t)_s$ by (3). If $0 \neq x \in I_t$, then $xU \in I_s$, and hence $xU = aU$ for some $0 \neq a \in I$ or $x \in aD \subseteq I$. Hence $I_t \subseteq I$, and thus $I_t = I$.

(5) Let $x \in K^*$, and suppose that $x \in (I_s)_r$. Then $x = \sum a_ib_i$ for some $b_i \in D^*$ and $a_i \in K^*$ with $a_iU \in I_s$; so $xU \in [\{a_iU\}] \subseteq I_s$ or $xU = aU$ for some $0 \neq a \in I$. Hence $x \in I$. Clearly, $I \subseteq (I_s)_r$, and thus $(I_s)_r = I$. Next, if $a \in D^*$, then $(I_s)_r = I$ implies "$a \in I \Leftrightarrow aU \in I_s$", and thus I is prime if and only if I_s is prime.

(6) Let $0 \neq x \in I_1I_2$. Then $x = \sum a_ib_i$ for some $0 \neq a_i \in I_1$ and $0 \neq b_i \in I_2$; hence $x \in (\{a_ib_i\})_v$, and by (1) $xU \in [\{a_ib_iU\}]_v \subseteq ([\{a_iU\}] + [\{b_iU\}])_v \subseteq ((I_1)_s + (I_2)_s)_t$. Hence $(I_1I_2)_s \subseteq ((I_1)_s + (I_2)_s)_t$, and thus $((I_1I_2)_s)_t \subseteq ((I_1)_s + (I_2)_s)_t$. Conversely, note that $(I_1)_s + (I_2)_s \subseteq (I_1I_2)_s$; so $((I_1)_s + (I_2)_s)_t \subseteq ((I_1I_2)_s)_t$. Thus $((I_1)_s + (I_2)_s)_t = ((I_1I_2)_s)_t$.

(7) By (2), (3) and (6), we have $\Gamma = ((II^{-1})_t)_s = ((II^{-1})_s)_t = ((I_s) + (I^{-1})_s)_t = ((I_s) + (I_s)^{-1})_t$. Thus $(II^{-1})_t = D$ if and only if $((I_s) + (I_s)^{-1})_t = \Gamma$. □

We next study the ideal J_r of D for a fractional ideal J of D^*/U.

PROPOSITION 4. *Let U be a group of units of D and $\Gamma = D^*/U$ be the semigroup. Let J be a fractional ideal of Γ.*
(1) $(J_r)^{-1} = (J^{-1})_r$; hence $(J_r)_v = (J_v)_r$.
(2) $(J_r)_t = (J_t)_r$.
(3) If J is a t-ideal, then J_r is a t-ideal and $(J_r)_s = J$.
(4) If J is t-invertible, then J_r is t-invertible.
(5) If J is a t-ideal, then J is prime if and only if J_r is prime.

Proof. (1) Let $x \in K^*$, and note that $xa \in D$ for all $a \in K^*$ with $aU \in J$ if and only if $xJ_r \subseteq D$. Hence $x \in (J_r)^{-1} \Rightarrow xJ_r \subseteq D \Rightarrow xU + J \subseteq \Gamma \Rightarrow xU \in J^{-1} \Rightarrow x \in (J^{-1})_r$. Conversely, if $x \in (J^{-1})_r$, then $x \in (a_1, \ldots, a_n)$ for some $a_i \in K^*$ with $a_iU \in J^{-1}$. Hence $xJ_r \subseteq (a_1, \ldots, a_n)J_r$, and since $a_iU + J \subseteq \Gamma$, we have $a_iJ_r \subseteq D$, and thus $xJ_r \subseteq D$ or $x \in (J_r)^{-1}$.

(2) Let $x \in K^*$. Suppose $x \in (J_r)_t$. Then $x \in (a_1, \ldots, a_n)_v$ for some $0 \neq a_1, \ldots, a_n \in J_r$. Note that $a_i \in (b_1, \ldots, b_m)$ for some $b_i \in K^*$ with $b_iU \in J$; so replacing a_i with $\{b_j\}$, we may assume that $a_iU \in J$. Hence by Proposition 3(1), $xU \in [a_1U, \ldots, a_nU]_v \subseteq J_t$, and thus $x \in (J_t)_r$. Conversely, $x \in (J_t)_r \Rightarrow x \in (c_1, \ldots, c_k)$ for some $c_i \in K^*$ with $c_iU \in J_t$ $\Rightarrow xU \in [c_1U, \ldots, c_kU]_v \subseteq (J_t)_t = J_t$ by Proposition 3(1) $\Rightarrow x \in (J_t)_r$.

(3) By (2), J_r is a t-ideal. Next, if $xU \in (J_r)_s$, then $x \in J_r$; hence $x \in (a_1, \ldots, a_n)$ for some $a_i \in K^*$ with $a_iU \in J$. Thus by Proposition 3(1), $xU \in [a_1U, \ldots, a_nU]_v \subseteq J_t = J$. Clearly, $J \subseteq (J_r)_s$, and thus $J = (J_r)_s$.

(4) Clearly, $(J + J^{-1})_r \subseteq (J_r)(J_r)^{-1}$; so $(J + J^{-1})_r \subseteq (J_r)(J^{-1})_r$ by (1). Conversely, if $x \in (J_r)(J^{-1})_r$, then $x = \sum a_ib_i$ for some $a_i \in J_r$ and $b_i \in (J^{-1})_r$; so $x \in (\{a_ib_i\}) = [\{a_ib_iU\}]_r \subseteq (J + J^{-1})_r$. Hence $(J + J^{-1})_r = (J_r)(J^{-1})_r$, and thus $D = (\Gamma)_r = ((J + J^{-1})_t)_r = ((J_r)(J^{-1})_r)_t$ by (2).

(5) By Proposition 3(4), J_r is a prime ideal if and only if $(J_r)_s$ is a prime ideal. Thus by (3), J_r is prime if and only if J is prime. □

REMARK 5. Let U be a group of units of D, and let $\Gamma = D^*/U$ be the semigroup.

(1) Let a, b be nonzero nonunits of D such that $(a, b) = D$ (for example, D has at least two maximal ideals). Let $J = [aU, bU]$, then $J_r = D$ and $(J_r)_s = \Gamma$. Thus the (3) and (5) of Propositon 4 does not hold if J is

not a t-ideal. For Proposition 4(4), note that if J is t-invertible, then J_r is t-invertible, and hence $(J_r)_s$ is t-invertible by Proposition 3(7). Thus if J is a t-ideal, then J is t-invertible if and only if J_r is t-invertible.

(2) Let $Div(D)$ (resp., $Div(D^*/U)$) be the semigroup of fractional t-ideals of D (resp., D^*/U) under $I_1 * I_2 = (I_1I_2)_t$ (resp., $J_1 * J_2 = (I_1 + J_2)_t$). Propositions 3 and 4 show that the map $\pi : Div(D) \to Div(D^*/U)$, given by $I \to I_s$, is a semigroup isomorphism. Also, Propositions 3(5) and 4(5) show that the restriction of π to t-Spec(D) is a bijection from t-Spec(D) into t-Spec(D^*/U).

We next study the relation between $Cl(D)$ and $Cl(D^*/U)$. Set $\Gamma = D^*/U$, and let $cl(I_1), cl(I_2) \in Cl(D)$. Note that if $cl(I_1) = cl(I_2)$, then $I_1 = xI_2$ for some $x \in K^*$; so $(I_1)_s = (xI_2)_s = xU + (I_2)_s$. Note also that $(I_1)_s$ and $(xI_2)_s$ are t-invertible t-ideals by Proposition 3(7), hence $cl((I_1)_s) = cl((I_2)_s)$. Thus the map $\varphi : Cl(D) \to Cl(\Gamma)$, given by $cl(I) \to cl(I_s)$, is well-defined. We next show $Cl(D) = Cl(D^*/U)$, which means that φ is a group isomorphism.

Corollary 6. *$Cl(D) = Cl(D^*/U)$.*

Proof. We first show that φ is a group homomorphism.
Let $cl(I_1), cl(I_2) \in Cl(D)$, and note that $((I_1I_2)_t)_s = ((I_1I_2)_s)_t = ((I_1)_s + (I_2)_s)_t$ by Proposition 3(3) and (6). Hence $\varphi(cl((I_1I_2)_t)) = cl(((I_1I_2)_t)_s) = cl((I_1)_s) + cl((I_2)_s) = \varphi(cl((I_1)_s) + \varphi(cl((I_1)_s)$.

Next, if $I_s = aU + \Gamma$, then $I = (I_s)_r = (aU + \Gamma)_r = aD$ by Proposition 3(5). Since φ is a homomorphism, φ is injective. Finally, we show that φ is surjective, and hence φ is a group isomorphism. To do this, let $cl(J) \in \Gamma$, where J is a t-invertible t-ideal of Γ. Then J_r is a t-invertible t-ideal of D such that $(J_r)_s = J$ by Proposition 4(3) and (4). Hence $cl(J_r) \in Cl(D)$ such that $\varphi(cl(J_r)) = cl(J)$. □

Let D be an integrally closed domain, and assume that D is not a valuation domain. Then $G = K^*/U(D)$ is totally ordered by Lemma 1(3) and [4, Corollary 3.4] but $G(D)$, the group of divisibility of D, is not totally ordered. Thus the order of $G(D)$ is different from that of G.

Corollary 7. *Let $\Gamma = D^*/U$ be the semigroup.*
(1) D is a PvMD if and only if Γ is a PvMS.
(2) D is a GCD-domain if and only if Γ is a GCD-semigroup.
(3) D is a Mori domain if and only if Γ is a Mori semigroup.

(4) (cf. [5, Theorem 23.4]) *D is a Krull domain if and only if Γ is a Krull semigroup.*
(5) D is a factorial domain if and only if Γ is a factorial semigroup.

Proof. (1) Suppose that D is a PvMD, and let J be a finite type t-ideal of Γ, i.e., $J = [a_1U, \ldots, a_nU]_v$ for some $a_iU \in \Gamma$. Then $J_r = (\{a_i\})_v$ by Proposition 4(1); hence J_r is t-ivertible and thus $J = (J_r)_s$ is t-invertible by Propositions 3(7) and 4(3). Thus Γ is a PvMS. Conversely, assume that Γ is a PvMS, and let $I = (b_1, \ldots, b_m)_v$ for $b_i \in D^*$. Then $I_s = [b_1U, \ldots, b_mU]_v$ by Proposition 3. Hence I_s is t-invertible, and thus $I = (I_s)_r$ is t-invertible by Proposition 3(4) and (7). Thus D is a PvMD.

(2) This is an immediate consequence of (1) and Corollary 6 because a PvMD D (resp., PvMS Γ) is a GCD-domain (resp., GCD-semigroup) if and only if $Cl(D) = 0$ (resp., $Cl(\Gamma) = 0$).

(3) This is an immediate consequence of Proposition 3(3) and (5) and Proposition 4(2) and (3).

(4) This can be proved by the same arguement as in the proof of (1) because each t-ideal of Krull domains and Krull semigroups is of finite type.

(5) Note that a Krull domain D (resp., Krull semigroup Γ) is factorial if and only if $Cl(D) = 0$ (resp., $Cl(\Gamma) = 0$). Thus the result is an immediate consequence of (4) and Corollary 6. □

Let Γ be a commutative cancellative semigroup, and let $D[\Gamma]$ be the semigroup ring of Γ over D. It is known that Γ is torsion-free if and only if $D[\Gamma]$ is an integral domain [4, Theorem 8.1] and that $D[\Gamma]$ is integrally closed if and only if D and Γ are integrally closed [4, Corollary 12.11].

Lemma 8. *Let $\Gamma = D^*/U(D)$ be the semigroup. If D is integrally closed, then Γ is torsion-free, and hence $D[\Gamma]$ is an integrally closed domain with $Cl(D[\Gamma]) = Cl(D) \oplus Cl(D)$.*

Proof. For $x \in K^*$, assume that $x^nU(D) = U(D)$ for an integer $n \geq 1$. Then $x^n \in U(D) \subseteq D$, and since D is integrally closed, $x \in D$. Also, $xx^{n-1} = x^n \in U(D)$ implies $x \in U(D)$. Hence $xU(D) = U(D)$. Thus Γ is torsion-free.

Next, it is clear that Γ is integrally closed; hence $D[\Gamma]$ is integrally closed and $Cl(D[\Gamma]) = Cl(D) \oplus Cl(D)$ by Corollary 6 and [2, Corollary 2.11]. □

Corollary 9. *The following statements are equivalent for* $\Gamma = D^*/U(D)$.
(1) D is a PvMD (resp., GCD-domain, Krull domain, factorial domain).
(2) $D[\Gamma]$ is a PvMD (resp., GCD-domain, Krull domain, factorial domain).
(3) $K[\Gamma]$ is a PvMD (resp., GCD-domain, Krull domain, factorial domain).
(4) $D[G]$ is a PvMD (resp., GCD-domain, Krull domain, factorial domain).

Proof. The PvMD and Krull domain cases.

(1) $\Leftrightarrow$ (2) and (1) $\Leftrightarrow$ (3) These follow directly from Corollary 7 and [1, Proposition 6.5] (resp., [4, Theorem 15.6]). (2) $\Rightarrow$ (4) This follows because $D[G] = D[\Gamma]_N$, where $N = \{X^\alpha | \alpha \in \Gamma\}$. (4) $\Rightarrow$ (1) This follows because $D[G] \cap K = D$.

The GCD-domain and factorial domains cases are immediate consequences of Lemma 8 and the PvMD and Krull domain cases because D is a GCD-domain (resp., factorial domain) if and only if D is a PvMD (resp., Krull domain) and $Cl(D) = 0$. □

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