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RESULTS ON FUZZY r-MINIMAL SEMICOMPACTNESS ON FUZZY MINIMAL SPACES

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Abstract. We introduce the concept of fuzzy co-r-M-semicontinuous, and investigate the relationships between fuzzy co-r-M-semicontinuous mappings and several types of fuzzy r-minimal semicompactness.

1. Introduction

The concept of fuzzy set was introduced by Zadeh [8]. Chang [1] defined fuzzy topological spaces using fuzzy sets. In [4], Ramadan introduced smooth topological spaces which are a generalization of fuzzy topological spaces. In [6], we introduced the concept of fuzzy r-minimal space which is an extension of the smooth topological space. The concepts of fuzzy r-minimal open sets and fuzzy r-M-continuous mappings were also introduced and studied. In [3], we introduced the concepts of fuzzy r-minimal semiopen sets and fuzzy r-M-semicontinuous mappings, which are generalizations of fuzzy r-minimal open sets and fuzzy r-M-continuous mappings, respectively. Yoo et al. introduced the concepts of fuzzy r-minimal compactness, almost fuzzy r-minimal compactness and nearly fuzzy r-minimal compactness on fuzzy r-minimal spaces in [7]. We introduced and studied the concepts of fuzzy r-minimal semicompact, almost fuzzy r-minimal semicompact, nearly fuzzy r-minimal semicompact on fuzzy r-minimal spaces in [5]. In this paper, we introduce the concept of fuzzy co-r-M-semicontinuous, and investigate the relationships between fuzzy co-r-M-semicontinuous mappings and several types of fuzzy r-minimal semicompactness.

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2. Preliminaries

Let I be the unit interval $[0,1]$ of the real line. A member A of I^X is called a *fuzzy set* [7] of X. By $\tilde{0}$ and $\tilde{1}$, we denote constant maps on X with value 0 and 1, respectively. For any $A \in I^X$, A^c denotes the complement $\tilde{1} - A$. All other notations are standard notations of fuzzy set theory.

An *fuzzy point* x_α in X is a fuzzy set x_α defined as follows

$$x_\alpha(y) = \begin{cases} \alpha \text{ if } y = x \\ 0 \text{ if } y \neq x. \end{cases}$$

Let $f : X \to Y$ be a function and $A \in I^X$ and $B \in I^Y$. Then $f(A)$ is a fuzzy set in Y, defined by

$$f(A)(y) = \begin{cases} \sup_{z \in f^{-1}(y)} A(z), & \text{if } f^{-1}(y) \neq \emptyset, \\ 0, & \text{otherwise}, \end{cases}$$

for $y \in Y$ and $f^{-1}(B)$ is a fuzzy set in X, defined by $f^{-1}(B)(x) = B(f(x)), x \in X$.

A *smooth topology* [2,5] on X is a map $\mathcal{T} : I^X \to I$ which satisfies the following properties:

1. $\mathcal{T}(\tilde{0}) = \mathcal{T}(\tilde{1}) = 1$.
2. $\mathcal{T}(A_1 \cap A_2) \geq \mathcal{T}(A_1) \wedge \mathcal{T}(A_2)$.
3. $\mathcal{T}(\cup A_i) \geq \wedge \mathcal{T}(A_i)$.

The pair $(X, \mathcal{T})$ is called a *smooth topological space.*

Let X be a nonempty set and $r \in (0,1] = I_0$. A fuzzy family $\mathcal{M} : I^X \to I$ on X is said to have a *fuzzy r-minimal structure* [6] if the family

$$\mathcal{M}_r = \{A \in I^X \mid \mathcal{M}(A) \geq r\}$$

contains $\tilde{0}$ and $\tilde{1}$.

Then the $(X, \mathcal{M})$ is called a *fuzzy r-minimal space* [6] (simply, fuzzy r-FMS). Every member of $\mathcal{M}_r$ is called a *fuzzy r-minimal open* set. A fuzzy set A is called a *fuzzy r-minimal closed* set if the complement of A is a fuzzy r-minimal open set.

Let $(X, \mathcal{M})$ be an r-FMS and $r \in I_0$. The fuzzy r-minimal closure of A, denoted by $mC(A,r)$, is defined as

$$mC(A,r) = \cap\{B \in I^X : \tilde{1} - B \in \mathcal{M}_r \text{ and } A \subseteq B\}.$$

The fuzzy r-minimal interior of A, denoted by $mI(A, r)$, is defined as

$$mI(A, r) = \cup\{B \in I^X : B \in \mathcal{M}_r \text{ and } B \subseteq A\}.$$

THEOREM 2.1 ([6]). Let $(X, \mathcal{M})$ be an r-FMS and $A, B \in I^X$.

(1) $mI(A, r) \subseteq A$ and if A is a fuzzy r-minimal open set, then $mI(A, r) = A$.

(2) $A \subseteq mC(A, r)$ and if A is a fuzzy r-minimal closed set, then $mC(A, r) = A$.

(3) If $A \subseteq B$, then $mI(A, r) \subseteq mI(B, r)$ and $mC(A, r) \subseteq mC(B, r)$.

(4) $mI(A, r) \cap mI(B, r) \supseteq mI(A \cap B, r)$ and $mC(A, r) \cup mC(B, r) \subseteq mC(A \cup B, r)$.

(5) $mI(mI(A, r), r) = mI(A, r)$ and $mC(mC(A, r), r) = mC(A, r)$.

(6) $\tilde{1} - mC(A, r) = mI(\tilde{1} - A, r)$ and $\tilde{1} - mI(A, r) = mC(\tilde{1} - A, r)$.

Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then a fuzzy set A is called a *fuzzy r-minimal semiopen set* [3] in X if

$$A \subseteq mC(mI(A, r), r).$$

A fuzzy set A is called a *fuzzy r-minimal semiclosed* set if the complement of A is fuzzy r-minimal semiopen.

We showed that any union of fuzzy r-minimal semiopen sets is fuzzy r-minimal semiopen [3].

For $A \in I^X$, $msC(A, r)$ and $msI(A, r)$, respectively, are defined as the following:

$$msC(A, r) = \cap\{F \in I^X : A \subseteq F, \quad F \text{ is fuzzy } r\text{-minimal semiclosed}\}$$

$$msI(A, r) = \cup\{U \in I^X : U \subseteq A, U \text{ is fuzzy } r\text{-minimal semiopen }\}.$$

THEOREM 2.2 ([3]). Let $(X, \mathcal{M})$ be an r-FMS and $A \in I^X$. Then

(1) $msI(A, r) \subseteq A \subseteq msC(A, r)$.

(2) If $A \subseteq B$, then $msI(A, r) \subseteq msI(B, r)$ and $msC(A, r) \subseteq msC(B, r)$.

(3) A is fuzzy r-minimal semiopen iff $msI(A, r) = A$.

(4) F is fuzzy r-minimal semiclosed iff $msC(F, r) = F$.

(5) $msI(msI(A, r), r) = msI(A, r)$ and $msC(msC(A, r), r) = msC(A, r)$.

(6) $msC(\tilde{1} - A, r) = \tilde{1} - msI(A, r)$ and $msI(\tilde{1} - A, r) = \tilde{1} - msC(A, r)$.

3. Main results

DEFINITION 3.1. Let $(X,\mathcal{M})$ and $(Y,\mathcal{N})$ be two r-FMS's. Then $f : X\to Y$ is said to be *fuzzy co-r-M-semicontinuous* if for every fuzzy r-minimal semiopen set V, $f^{-1}(V)$ is fuzzy r-minimal open in Y.

THEOREM 3.2. Let $f : X\to Y$ be a mapping on two r-FMS's $(X,\mathcal{M})$ and $(Y,\mathcal{N})$.
(1) f is fuzzy co-r-M-continuous.
(2) $f^{-1}(B)$ is a fuzzy r-minimal closed set for each fuzzy r-minimal semiclosed set B in Y.
(3) $f(mC(A,r))\subseteq msC(f(A),r)$ for $A\in I^X$.
(4) $mC(f^{-1}(B),r)\subseteq f^{-1}(msC(B,r))$ for $B\in I^Y$.
(5) $f^{-1}(msI(B,r))\subseteq mI(f^{-1}(B),r)$ for $B\in I^Y$.
Then (1) $\Leftrightarrow$ (2) $\Rightarrow$ (3) $\Leftrightarrow$ (4) $\Leftrightarrow$ (5).

Proof. (1) $\Leftrightarrow$ (2) Obvious.
(2) $\Rightarrow$ (3) For $A\in I^X$,

$$\begin{aligned}
&f^{-1}(msC(f(A),r))\\
&=f^{-1}(\cap\{F\in I^Y : f(A)\subseteq F \text{ and } F \text{ is fuzzy } r\text{-minimal semiclosed}\})\\
&=\cap\{f^{-1}(F)\in I^X : A\subseteq f^{-1}(F) \text{ and } f^{-1}(F) \text{ is fuzzy } r\text{-minimal closed}\}\\
&\supseteq\cap\{K\in I^X : A\subseteq K \text{ and } K \text{ is fuzzy } r\text{-minimal closed}\}\\
&=mC(A,r).
\end{aligned}$$

Hence $f(mC(A,r))\subseteq msC(f(A),r)$.
(3) $\Rightarrow$ (4) Let $B\in I^Y$. Then $f(B)\in I^X$ and from (3), it follows that

$$f(mC(f^{-1}(B),r))\subseteq msC(f(f^{-1}(B)),r)\subseteq msC(B,r).$$

Hence we get (4). Similarly, we get (4) $\Rightarrow$ (3).
(4) $\Leftrightarrow$ (5) It is obvious. □

EXAMPLE 3.3. Let $X=I$ and let A,B be fuzzy sets defined as follows:

$$A(x)=x,\quad x\in I;$$
$$B(x)=-x,\quad x\in I.$$

Define

$$\mathcal{M}(\sigma)=\begin{cases}\frac{1}{2}, & \text{if } \sigma=\tilde{0},\tilde{1},\\ \frac{2}{3}, & \text{if } \sigma=A,B,\\ 0, & \text{otherwise.}\end{cases}$$

Define

$$\mathcal{N}(\sigma) = \begin{cases} \frac{2}{3}, & \text{if } \sigma = A, B, \\ \frac{1}{2}, & \text{if } \sigma = A \cup B, \tilde{0}, \tilde{1}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $f : (X, \mathcal{M}) \to (X, \mathcal{N})$ be the identity mapping. Then f is satisfies (3) of Theorem 3.2 but it is not fuzzy co-$\frac{1}{2}$-M-continuous since $A \cap B$ is fuzzy $\frac{1}{2}$-minimal semiopen in Y but not fuzzy $\frac{1}{2}$-minimal open in X.

THEOREM 3.4. Let $f : X \to Y$ be a mapping on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the statements are equivalent:

(1) If for every fuzzy point x_α and each fuzzy r-minimal semiopen set V of $f(x_\alpha)$, there exists an r-minimal open set U of x_α such that $f(U) \subseteq V$.

(2) $f^{-1}(msI(B, r)) \subseteq mI(f^{-1}(B), r)$ for $B \in I^Y$.

Proof. (1) $\Rightarrow$ (2) Let $B \in I^Y$ and $x_\alpha \in f^{-1}(msI(B, r))$. Then there exists a fuzzy r-minimal semiopen set V of $f(x_\alpha)$ such that $V \subseteq B$. By hypothesis, there exists a fuzzy r-minimal open U_{x_α} containing x_α such that $f(U_{x_\alpha}) \subseteq V \subseteq B$. Then from the definition of fuzzy r-minimal interior operator, $x_\alpha \in mI(f^{-1}(B), r)$. Hence, we have $f^{-1}(msI(B, r)) \subseteq mI(f^{-1}(B), r)$.

(2) $\Rightarrow$ (1) Let x_α be a fuzzy point in X and V a fuzzy r-minimal semiopen set of $f(x_\alpha)$. Since $f^{-1}(V)$ is fuzzy r-minimal open, $x_\alpha \in f^{-1}(V) = mI(f^{-1}(V), r)$. Thus there exists a fuzzy r-minimal open set U such that $x_\alpha \in U \subseteq f^{-1}(V)$. Hence we have the statement (1).

□

Let X be a nonempty set and $\mathcal{M} : I^X \to I$ a fuzzy family on X. The fuzzy family $\mathcal{M}$ is said to have the property $(\mathcal{U})$ [6] if for $A_i \in \mathcal{M}$ $(i \in J)$,

$$\mathcal{M}(\cup A_i) \geq \wedge \mathcal{M}(A_i).$$

THEOREM 3.5 ([6]). Let $(X, \mathcal{M})$ be an r-FMS with the property $(\mathcal{U})$. Then

(1) $mI(A, r) = A$ if and only if $A \in \mathcal{M}_r$ for $A \in I^X$.

(2) $mC(A, r) = A$ if and only if $\tilde{1} - A \in \mathcal{M}_r$ for $A \in I^X$.

COROLLARY 3.6. Let $f : X \to Y$ be a mapping on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. If $\mathcal{M}$ has the property $(\mathcal{U})$, then the following are equivalent:

(1) f is fuzzy co-r-M-continuous.

(2) $f^{-1}(B)$ is a fuzzy r-minimal closed set, for each fuzzy r-minimal semiclosed set B in Y.
(3) $f(mC(A,r)) \subseteq msC(f(A),r)$ for $A \in I^X$.
(4) $mC(f^{-1}(B),r) \subseteq f^{-1}(msC(B,r))$ for $B \in I^Y$.
(5) $f^{-1}(msI(B,r)) \subseteq mI(f^{-1}(B),r)$ for $B \in I^Y$.

Definition 3.7. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two r-FMS's. Then a mapping $f : X \to Y$ is called *fuzzy r-M-semiopen* if for every $A \in \mathcal{M}_r$, $f(A)$ is fuzzy r-minimal semiopen.

We recall that: Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two r-FMS's. Then a mapping $f : X \to Y$ is called *fuzzy r-M-open* [6] if for every $A \in \mathcal{M}_r$, $f(A)$ is fuzzy r-minimal open.

Obviously the implication is obtained:

fuzzy r-M-open mapping $\Rightarrow$ fuzzy r-M-semiopen mapping

Example 3.8. Let $X = I$. Consider fuzzy sets A, B defined as follows:

$$A(x) = \frac{1}{2}x, \quad x \in I;$$

$$B(x) = -\frac{1}{2}(x-1), \quad x \in I.$$

Define

$$\mathcal{M}(\sigma) = \begin{cases} \frac{1}{2}, & \text{if } \sigma = A, B \\ \frac{2}{3}, & \text{if } \sigma = A \cup B, \tilde{0}, \tilde{1} \\ 0, & \text{otherwise.} \end{cases}$$

Define

$$\mathcal{N}(\sigma) = \begin{cases} \frac{2}{3}, & \text{if } \sigma = A, B \\ \frac{1}{2}, & \text{if } \sigma = \tilde{0}, \tilde{1} \\ 0, & \text{otherwise.} \end{cases}$$

Let $f : (X, \mathcal{M}) \to (X, \mathcal{N})$ be the identity mapping. Then f is fuzzy $\frac{1}{2}$-M-semiopen but not fuzzy $\frac{1}{2}$-M-open.

Theorem 3.9. Let $f : X \to Y$ be a mapping on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following are equivalent:
(1) f is fuzzy r-M-semiopen.
(2) $f(mI(A),r) \subseteq msI(f(A),r)$ for $A \in I^X$.
(3) $mI(f^{-1}(B),r) \subseteq f^{-1}(msI(B),r)$ for $B \in I^Y$.

Proof. (1) ⇒ (2) For $A \in I^X$,

$f(mI(A), r) = f(\cup\{B \in I^X : B \subseteq A, B$ is fuzzy r-minimal open$\})$

$= \cup\{f(B) \in I^Y : f(B) \subseteq f(A), f(B)$ is fuzzy r-minimal semiopen$\}$

$\subseteq \cup\{U \in I^Y : U \subseteq f(A), U$ is fuzzy r-minimal semiopen$\}$

$= msI(f(A), r)$.

Hence $f(mI(A), r) \subseteq msI(f(A), r)$.

(2) ⇒ (1) For a fuzzy r-minimal open A, from Theorem 2.1 and hypothesis,

$$f(A) = f(mI(A), r) \subseteq msI(f(A), r).$$

From Theorem 2.2, $f(A)$ is fuzzy r-minimal semiopen and hence f is fuzzy r-M-semiopen.

(2) ⇒ (3) For $B \in I^Y$, from (2) it follows that

$$f(mI(f^{-1}(B), r)) \subseteq msI(f(f^{-1}(B)), r) \subseteq msI(B, r).$$

Hence we get (3).

Similarly, we get (3) ⇒ (2).

□

Definition 3.10. Let $(X, \mathcal{M})$ and $(Y, \mathcal{N})$ be two r-FMS's. Then a mapping $f : X \to Y$ is called *fuzzy r-M-semiclosed* if for every fuzzy r-minimal closed set A in X, $f(A)$ is a fuzzy r-minimal semiclosed set in Y.

Theorem 3.11. Let $f : X \to Y$ be a mapping on two r-FMS's $(X, \mathcal{M})$ and $(Y, \mathcal{N})$. Then the following are equivalent:

(1) f is fuzzy r-M-semiclosed.

(2) $msC(f(A), r) \subseteq f(mC(A, r))$ for $A \in I^X$.

(3) $f^{-1}(msC(B, r)) \subseteq mC(f^{-1}(B), r)$ for $B \in I^Y$.

Proof. It is similar to Theorem 3.3. □

We recall the concepts of several types of fuzzy r-minimal compactness introduced in [7]. Let $(X, \mathcal{M})$ be an r-FMS and $\mathcal{A} = \{A_i \in I^X : i \in J\}$. $\mathcal{A}$ is called a *fuzzy r-minimal cover* if $\cup\{A_i : i \in J\} = \tilde{\mathbf{1}}$. It is a *fuzzy r-minimal open cover* if each A_i is a fuzzy r-minimal open set. A subcover of a fuzzy r-minimal cover $\mathcal{A}$ is a subfamily of it which also is a fuzzy r-minimal cover. A fuzzy set A in X is said to be

(1) *fuzzy r-minimal compact* if every fuzzy r-minimal open cover $\mathcal{A} = \{A_i \in \mathcal{M}_r : i \in J\}$ of A has a finite subcover;

(2) *almost fuzzy r-minimal compact* (resp., *nearly fuzzy r-minimal compact*) if for every fuzzy r-minimal open cover $\mathcal{A} = \{A_i \in I^X : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} mC(A_k, r)$ (resp., $A \subseteq \cup_{k \in J_0} mI(mC(A_k, r), r))$.

DEFINITION 3.12 ([4]). Let $(X, \mathcal{M})$ be an r-FMS. A fuzzy set A in X is said to be

(1)*fuzzy r-minimal semicompact* (resp., if every fuzzy r-minimal semiopen cover $\mathcal{A} = \{A_i \in \mathcal{M}_r : i \in J\}$ of A has a finite subcover;

(2) *almost fuzzy r-minimal semicompact* if for every fuzzy r-minimal semiopen cover $\mathcal{A} = \{A_i \in I^X : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} msC(A_k, r)$;

(3) *nearly fuzzy r-minimal semicompact* if for every fuzzy r-minimal semiopen cover $\mathcal{A} = \{A_i : i \in J\}$ of A, there exists $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} msI(msC(A_k, r), r)$.

THEOREM 3.13. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy co-r-M-semicontinuous mapping on two r-FMS's. If A is a fuzzy r-minimal compact set, then $f(A)$ is fuzzy r-minimal semicompact.

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal semiopen cover of $f(A)$ in Y. Then since f is a fuzzy co-r-M-semicontinuous mapping, $\{f^{-1}(B_i) : i \in J\}$ is a fuzzy r-minimal open cover of A in X. Since A is fuzzy r-minimal compact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} f^{-1}(B_k)$. It implies $f(A) \subseteq \cup_{k \in J_0} B_k$ for the finite subset J_0 of J, and hence $f(A)$ is fuzzy r-minimal semicompact. □

THEOREM 3.14. Let $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be a fuzzy co-r-M-semicontinuous mapping on two r-FMS's. If A is an almost fuzzy r-minimal compact set, then $f(A)$ is almost fuzzy r-minimal semicompact.

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal semiopen cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is a fuzzy r-minimal open cover of A in X. Since A is almost fuzzy r-minimal compact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} mC(f^{-1}(B_k), r)$. It follows

$$\begin{aligned} \cup_{k \in J_0} mC(f^{-1}(B_k, r)) &\subseteq \cup_{k \in J_0} f^{-1}(msC(B_k, r)) \\ &= f^{-1}(\cup_{k \in J_0} msC(B_k, r)). \end{aligned}$$

So $f(A) \subseteq \cup_{k \in J_0} msC(B_k, r)$ and $f(A)$ is almost fuzzy r-minimal semicompact. □

THEOREM 3.15. Let a mapping $f : (X, \mathcal{M}) \to (Y, \mathcal{N})$ be fuzzy co-r-M-semicontinuous and fuzzy r-M-semiopen on two r-FMS's. If A is a nearly fuzzy r-minimal compact set, then $f(A)$ is nearly fuzzy r-minimal semicompact.

Proof. Let $\{B_i \in I^Y : i \in J\}$ be a fuzzy r-minimal semiopen cover of $f(A)$ in Y. Then $\{f^{-1}(B_i) : i \in J\}$ is a fuzzy r-minimal open cover of A in X. Since X is nearly fuzzy r-minimal compact, there exists a finite subset $J_0 = \{j_1, j_2, \cdots, j_n\} \subseteq J$ such that $A \subseteq \cup_{k \in J_0} mI(mC(f^{-1}(B_k), r), r)$. It follows

$$\begin{aligned} f(A) &\subseteq \cup_{k \in J_0} f(mI(mC(f^{-1}(B_k), r), r)) \\ &\subseteq \cup_{k \in J_0} msI(f(mC(f^{-1}(B_k), r)), r) \\ &\subseteq \cup_{k \in J_0} msI(f(f^{-1}(msC(B_k, r))), r) \\ &\subseteq \cup_{k \in J_0} msI(msC(B_k, r), r). \end{aligned}$$

Hence $f(A)$ is nearly fuzzy r-minimal semicompact. □

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