Korean J. Math. **18** (2010), No. 1, pp. 11–19

FIXED DEGREE THEOREMS FOR FUZZY MAPPINGS IN SYMMETRIC SPACES

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Abstract. In this paper, several common fixed degree theorems of a sequence of fuzzy mappings defined on symmetric spaces are established.

1. Introduction

In [6], the author introduced the concept of fixed degree for fuzzy mappings in complete metric spaces and proved some fixed degree theorems which are generalizations and unifications of some results of [2,3,4,12].

In [10], the authors obtained some theorems about common fixed degree of a sequence of fuzzy mappings in probabilistic metric spaces. The results of [10] are generalizations and unifications of some results of [2,5-9,13,15,16,17,20,21]. In [14], the author gave a generalization of the result of [10].

On the other hand, in [1,18,19], the authors gave some fixed point theorems in symmetric spaces. These theorems should be of interest to analysists. Recently, in [11], the authors studied some axioms for symmetric spaces and their relationships, and gave some examples. They proved some common fixed point theorems on symmetric spaces by using those axioms.

In this paper, we obtain some theorems about common fixed degree of a sequence of fuzzy mappings in symmetric spaces.

Received October 12, 2009. Revised February 6, 2010. Accepted Febraury 10, 2010.

2000 Mathematics Subject Classification: 54C40, 14E20, 46E25, 20C20.

Key words and phrases: fixed degree, common fixed point, symmetric space.

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2. Preliminaries

A *symmetric* on a set X is a function $d : X \times X \to [0, \infty)$ satisfying the following conditions:

(i) $d(x, y) = 0$ if and only if $x = y$ for all $x, y \in X$,

(ii) $d(x, y) = d(y, x)$ for all $x, y \in X$.

Let d be a symmetric on a set X. For $x \in X$ and $\epsilon > 0$, let $B(x, \epsilon) = \{y \in X : d(x, y) < \epsilon\}$. A topology $\tau(d)$ on X is defined as follows: $U \in \tau(d)$ if and only if for each $x \in U$, there exists an $\epsilon > 0$ such that $B(x, \epsilon) \subset U$. A subset S of X is a neighborhood of $x \in X$ if there exists $U \in \tau(d)$ such that $x \in U \subset S$. A symmetric d is a *semi-metric* if for each $x \in X$ and each $\epsilon > 0$, $B(x, \epsilon)$ is a neighborhood of x in the topology $\tau(d)$.

A *symmetric*(resp., *semimetric*) *space* (X, d) is a topological space whose topology $\tau(d)$ on X is induced by the symmetric(resp., semimetric) d.

Note that for a sequence $\{x_n\}$ in semimetric space (X, d) and $x \in X$, $lim_{n\to\infty} d(x_n, x) = 0$ if and only if $lim_{n\to\infty} x_n = x$ with respect to $\tau(d)$.

A symmetric(semimetric) space (X, d) is *summable-complete*(shortly, *s-complete*) if for every sequence $\{x_n\}$ with $\sum_{n=0}^{\infty} d(x_n, x_{n+1}) < \infty$, there exists an $x \in X$ such that $lim_{n\to\infty} d(x_n, x) = 0$.

The difference of a symmetric and a metric comes from the triangle inequality. Actually a symmetric space need not be Housdorff. In order to obtain fixed degree theorems on a symmetric space, we need the following axiom (C.C)[11].

(C.C) For a sequence $\{x_n\}$ in a symmetric space (X, d) and $x, y \in X$, $lim_{n\to\infty} d(x_n, x) = 0$ implies $lim_{n\to\infty} d(x_n, y) = d(x, y)$.

For a symmetric(semimetric) space (X, d), let $CB(X)$ be the collection of nonempty closed bounded subsets of X.

For all $A, B \in CB(X)$ and $x \in X$, let $d(x, A) = inf_{y\in A} d(x, y)$,

$H(A, B) = max\{sup_{x\in A} d(x, B), sup_{y\in B} d(A, y)\}$ and

$D(A, B) = sup_{x\in A} inf_{y\in B} d(x, y)$.

Note that $D(A, B) \leq H(A, B)$ for all $A, B \in CB(X)$.

A *fuzzy set* F in X is a function from X into $[0, 1]$ and $F(x)$ is called the grade of membership of $x \in X$ in F. For $\alpha \in (0, 1]$, the set $(F)_\alpha = \{x \in X : F(x) \geq \alpha\}$ is called the α-level set of a fuzzy set F.

We denote by $CBF(X)$ the collection of all fuzzy sets in X such that for any $F \in CBF(X)$ its each α-level set is a nonempty closed and bounded subset of X.

Recall that for fuzzy sets F and G in X, $F \subset G$ if and only if $F(x) \leq G(x)$ for all $x \in X$.

For each $x \in X$, we denote by $\{x\}$ the fuzzy set with a membership function equaling to the characteristic function of the singleton $\{x\}$ in X.

If T is a mapping from X into $CBF(X)$, then we say that T is a *fuzzy mapping* on X.

Let T be a fuzzy mapping and $\{T_i\}_{i=1}^{\infty}$ be a sequence of fuzzy mappings on a set X. For a $x_* \in X$, the value $(Tx_*)(x_*)$ is called the *fixed degree* of x_* for T, and $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) = inf_{i \geq 1}(T_i x_*)(x_*)$ is called the *common fixed degree* of x_* for $\{T_i\}_{i=1}^{\infty}$.

A point $x_* \in X$ is called a *fixed point* of T if $\{x_*\} \subset Tx_*$. A point $x_* \in X$ is called a *common fixed point* of $\{T_i\}_{i=1}^{\infty}$ if $\{x_*\} \subset \bigcap_{i=1}^{\infty} T_i x_*$.

Note that for a fuzzy mapping T and a sequence of fuzzy mappings $\{T_i\}_{i=1}^{\infty}$, $\{x_*\} \subset Tx_*$ if and only if $(Tx_*)(x_*) = 1$ and $\{x_*\} \subset \bigcap_{i=1}^{\infty} T_i x_*$ if and only if $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) = 1$. Thus the notion of fixed degree is a generalization of the notion of fixed point.

From now on, let $\phi : [0, \infty) \to [0, \infty)$ be a function satisfying the following conditions:

(ϕ1) ϕ is strictly increasing,

(ϕ2) $0 < \phi(t) < t$ for all $t > 0$,

(ϕ3) $\sum_{n=1}^{\infty} \phi^n(t) < \infty$ for all $t > 0$, where ϕ^n is n-th iteration of ϕ.

Note that $\phi(0) = 0$ and $lim_{n \to \infty} \phi^n(t) = 0$ for all $t \geq 0$.

3. Main results

THEOREM 3.1. *Let (X, d) be a s-complete symmetric(semimetric) space which satisfies (C.C), and let $\alpha : X \to (0, 1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*
for any $i, j \in \mathbb{N}$, $x, y \in X$ and $u \in (T_i x)_{\alpha(x)}$,

(1) $d(u, (T_j y)_{\alpha(y)}) \leq \phi(max\{d(x, y), d(x, (T_i x)_{\alpha(x)}), d(y, (T_j y)_{\alpha(y)})$.

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) \geq \alpha(x_*)$.

In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

Proof. Let $x_0 \in X$ and $x_1 \in (T_1x_0)_{\alpha(x_0)}$. Fix a real $c > 0$ with $d(x_0, x_1) < c$.

From (1) we have

$$\begin{aligned}
&d(x_1, (T_2x_1)_{\alpha(x_1)})\\
\leq&\phi(max\{d(x_0, x_1), d(x_0, (T_1x_0)_{\alpha(x_0)}), d(x_1, (T_2x_1)_{\alpha(x_1)}),\\
\leq&\phi(max\{d(x_0, x_1), d(x_1, (T_2x_1)_{\alpha(x_1)})\})\\
\leq&\phi(d(x_0, x_1))\\
<&\phi(c).
\end{aligned}$$

We can take $x_2 \in (T_2x_1)_{\alpha(x_1)}$ such that $d(x_1, x_2) < \phi(c)$.

Similarly, we have

$$\begin{aligned}
&d(x_2, (T_3x_2)_{\alpha(x_2)})\\
\leq&\phi(max\{d(x_1, x_2), d(x_1, (T_2x_1)_{\alpha(x_1)}), d(x_2, (T_3x_2)_{\alpha(x_2)}),\\
\leq&\phi(max\{d(x_1, x_2), d(x_2, (T_3x_2)_{\alpha(x_2)})\})\\
\leq&\phi(d(x_1, x_2))\\
<&\phi^2(c).
\end{aligned}$$

We can take $x_3 \in (T_3x_2)_{\alpha(x_2)}$ such that $d(x_2, x_3) < \phi^2(c)$.

Repeating the above procedure, we have a sequence $\{x_n\}$ in X such that

$x_{n+1} \in (T_{n+1}x_n)_{\alpha(x_n)}$ and $d(x_n, x_{n+1}) < \phi^n(c)$ for $n = 1, 2, 3, \cdots$.

Hence

$$lim_{n\to\infty} d(x_n, x_{n+1}) = 0$$

and

$$\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \sum_{n=1}^{\infty} \phi^n(c) < \infty.$$

By the s-completeness of (X, d), there exists an $x_* \in X$ such that $lim_{n\to\infty} d(x_n, x_*) = 0$.

Next, we show that $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) \geq \alpha(x_*)$.

From (1) we have

$$\begin{aligned}
&d(x_{n+1}, (T_ix_*)_{\alpha(x_*)})\\
&\leq \phi(max\{d(x_n, x_*), d(x_n, (T_{n+1}x_n)_{\alpha(x_n)}), d(x_*, (T_ix_*)_{\alpha(x_*)}),\\
&\leq \phi(max\{d(x_n, x_*), d(x_n, x_{n+1}), d(x_*, (T_ix_*)_{\alpha(x_*)}).
\end{aligned}$$

Letting $n \to \infty$ in above inequality and by using (C.C), we have $d(x_*, (T_i x_*)_{\alpha(x_*)}) \le \phi(d(x_*, (T_i x_*)_{\alpha(x_*)}))$.

Hence we have

$d(x_*, (T_i x_*)_{\alpha(x_*)}) \le \phi(d(x_*, (T_i x_*)_{\alpha(x_*)})) \le \phi^2(d(x_*, (T_i x_*)_{\alpha(x_*)}))$
$\le \cdots \le \phi^n(d(x_*, (T_i x_*)_{\alpha(x_*)})) \to 0$ as $n \to \infty$.

Thus $d(x_*, (T_i x_*)_{\alpha(x_*)}) = 0$ and $x_* \in (T_i x_*)_{\alpha(x_*)}$.

Since i was an arbitrary, we have $x_* \in (T_i x_*)_{\alpha(x_*)}$ for all $i \in \mathbb{N}$, and so

$(T_i x_*)(x_*) \ge \alpha(x_*)$ for all $i \in \mathbb{N}$. Hence,

$(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) = inf_{i \ge 1}(T_i x_*)(x_*) \ge \alpha(x_*)$.

If $\alpha(x_*) = 1$, then $x_* \in \bigcap_{i=1}^{\infty}(T_i x_*)_1$. Thus, $\{x_*\} \subset \bigcap_{i=1}^{\infty} T_i x_*$. □

Corollary 3.2. *Let (X, d) be a s-complete symmetric(semimetric) space which satisfies (C.C), and let $\alpha : X \to (0, 1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*
for any $i, j \in \mathbb{N}$, $x, y \in X$ and $u \in (T_i x)_{\alpha(x)}$, there exists $v \in (T_j y)_{\alpha(y)}$ such that $d(u, v) \le \phi(max\{d(x, y), d(x, (T_i x)_{\alpha(x)}), d(y, (T_j y)_{\alpha(y)})$.

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_i x_*)_{(x_*)} \ge \alpha(x_*)$.

In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

Corollary 3.3. *Let (X, d) be a s-complete symmetric(semimetric) space which satisfies (C.C), and let $\alpha : X \to (0, 1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*
for any $i, j \in \mathbb{N}$, $x, y \in X$,

$$D((T_i x)_{\alpha(x)}, (T_j y)_{\alpha(y)}) \le \phi(max\{d(x, y), d(x, (T_i x)_{\alpha(x)}), d(y, (T_j y)_{\alpha(y)}).$$

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_i x_*)_{(x_*)} \ge \alpha(x_*)$.

In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

Corollary 3.4. *Let (X, d) be a s-complete symmetric(semimetric) space which satisfies (C.C), and let $\alpha : X \to (0, 1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*
for any $i, j \in \mathbb{N}$, $x, y \in X$,

$$H((T_i x)_{\alpha(x)}, (T_j y)_{\alpha(y)}) \le \phi(max\ \{d(x, y), d(x, (T_i x)_{\alpha(x)}), d(y, (T_j y)_{\alpha(y)}).$$

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_i x_*)_{(x_*)} \ge \alpha(x_*)$.

In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

REMARK 3.5. In Theorem 3.1, if we have $\phi(t) = kt$, for some $k \in (0,1)$ and all $t \geq 0$, then the conclusion holds.

THEOREM 3.6. *Let (X, d) be a s-complete symmetric(semimetric) space which satisfies (C.C) and let $\alpha : X \to (0,1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings from X into $CBF(X)$ satisfying the following condition:*
there exist $a_1, a_2, a_3 \in (0,1)$ with $a_1 + a_2 + a_3 < 1$ such that for any $i, j \in \mathbb{N}$, $x, y \in X$ and $u \in (T_i x)_{\alpha(x)}$,

(2) $d(u, (T_j y)_{\alpha(y)}) \leq a_1 d(x, y) + a_2 d(x, (T_i x)_{\alpha(x)}) + a_3 d(y, (T_j y)_{\alpha(y)})$

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) \geq \alpha(x_*)$.
In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

Proof. As in proof of Theorem 3.1, let $x_0 \in X$ and $x_1 \in (T_1 x_0)_{\alpha(x_0)}$. Fix $c > 0$ with $d(x_0, x_1) < c$.

From (2) we have

$$\begin{aligned} & d(x_1, (T_2 x_1)_{\alpha(x_1)}) \\ \leq & a_1 d(x_0, x_1) + a_2 d(x_0, (T_1 x_0)_{\alpha(x_0)}) + a_3 d(x_1, (T_2 x_1)_{\alpha(x_1)}) \\ \leq & a_1 d(x_0, x_1) + a_2 d(x_0, x_1) + a_3 d(x_1, (T_2 x_1)_{\alpha(x_1)}). \end{aligned}$$

Hence we have

$$d(x_1, (T_2 x_1)_{\alpha(x_1)}) \leq \frac{a_1 + a_2}{1 - a_3} d(x_0, x_1) < \frac{a_1 + a_2}{1 - a_3} c.$$

We can take $x_2 \in (T_2 x_1)_{\alpha(x_1)}$ such that $d(x_1, x_2) < \frac{a_1 + a_2}{1 - a_3} c$.

Similarly, we have

$$\begin{aligned} & d(x_2, (T_3 x_2)_{\alpha(x_2)}) \\ \leq & a_1 d(x_1, x_2) + a_2 d(x_1, (T_2 x_1)_{\alpha(x_1)}) + a_3 d(x_2, (T_3 x_2)_{\alpha(x_2)}) \\ \leq & a_1 d(x_1, x_2) + a_2 d(x_1, x_2) + a_3 d(x_2, (T_3 x_2)_{\alpha(x_2)}). \end{aligned}$$

Hence we have
$d(x_2, (T_3 x_2)_{\alpha(x_2)}) \leq \frac{a_1 + a_2}{1 - a_3} d(x_1, x_2) < (\frac{a_1 + a_2}{1 - a_3})^2 c$.

We can take $x_3 \in (T_3 x_2)_{\alpha(x_2)}$ such that $d(x_2, x_3) < (\frac{a_1 + a_2}{1 - a_3})^2 c$.

Repeating the above procedure, we have a sequence $\{x_n\}$ in X such that

$$x_{n+1} \in (T_{n+1} x_n)_{\alpha(x_n)}$$

and

$$d(x_n, x_{n+1}) < (\frac{a_1 + a_2}{1 - a_3})^n c \ \ for \ \ n = 1, 2, 3, \cdots .$$

Thus

$$lim_{n\to\infty} d(x_n, x_{n+1}) = 0$$

and

$$\sum_{n=1}^{\infty} d(x_n, x_{n+1}) < \sum_{n=1}^{\infty} (\frac{a_1 + a_2}{1 - a_3})^n c < \infty.$$

By the s-completeness of (X, d), there exists an $x_* \in X$ such that $lim_{n\to\infty} d(x_n, x_*) = 0$.

We now show that $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) \geq \alpha(x_*)$.

From (2) we have

$d(x_{n+1}, (T_i x_*)_{\alpha(x_*)})$

$\leq a_1 d(x_n, x_*) + a_2 d(x_n, (T_{n+1} x_n)_{\alpha(x_n)}) + a_3 d(x_*, (T_i x_*)_{\alpha(x_*)})$, and so $d(x_{n+1}, (T_i x_*)_{\alpha(x_*)})$

$\leq a_1 d(x_n, x_*) + a_2 d(x_n, x_{n+1}) + a_3 d(x_*, (T_i x_*)_{\alpha(x_*)})$.

Letting $n \to \infty$ in above inequality and by using (C.C), we have

$$d(x_*, (T_i x_*)_{\alpha(x_*)}) \leq a_3 d(x_*, (T_i x_*)_{\alpha(x_*)}),$$

and so we have $d(x_*, (T_i x_*)_{\alpha(x_*)}) = 0$ and $x_* \in (T_i x_*)_{\alpha(x_*)}$.

By the arbitrariness of i, we have $x_* \in (T_i x_*)_{\alpha(x_*)}$ for all $i \in \mathbb{N}$, and so

$(T_i x_*)(x_*) \geq \alpha(x_*)$ for all $i \in \mathbb{N}$.

Hence $(\bigcap_{i=1}^{\infty} T_i x_*)(x_*) = inf_{i\geq 1}(T_i x_*)(x_*) \geq \alpha(x_*)$.

If $\alpha(x_*) = 1$ then $x_* \in \bigcap_{i=1}^{\infty} (T_i x_*)_1$. Thus $\{x_*\} \subset \bigcap_{i=1}^{\infty} T_i x_*$. □

Corollary 3.7. *Let (X, d) be a s-complete symmetric(semimetric) space which satisfies (C.C) and let $\alpha : X \to (0, 1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*

there exist $a_1, a_2, a_3 \in (0, 1)$ with $a_1 + a_2 + a_3 < 1$ such that for any $i, j \in \mathbb{N}$, $x, y \in X$ and $u \in (T_i x)_{\alpha(x)}$, there exists $v \in (T_j y)_{\alpha(y)}$ such that

$$d(u, v) \leq a_1 d(x, y) + a_2 d(x, (T_i x)_{\alpha(x)}) + a_3 d(y, (T_j y)_{\alpha(y)})$$

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_i x_*)_{(x_*)} \geq \alpha(x_*)$.

In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

Corollary 3.8. *Let (X,d) be a s-complete symmetric(semimetric) space which satisfies (C.C) and let $\alpha : X \to (0,1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*
there exist $a_1, a_2, a_3 \in (0,1)$ with $a_1 + a_2 + a_3 < 1$ such that for any $i, j \in \mathbb{N}$, $x, y \in X$,

$$D((T_ix)_{\alpha(x)}, (T_jy)_{\alpha(y)}) \leq a_1d(x,y)+a_2d(x,(T_ix)_{\alpha(x)})+a_3d(y,(T_jy)_{\alpha(y)})$$

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_ix_*)_{(x_*)} \geq \alpha(x_*)$.
In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

Corollary 3.9. *Let (X,d) be a S-complete symmetric(semimetric) space which satisfies (C.C) and let $\alpha : X \to (0,1]$ be a function. Suppose $\{T_i\}_{i=1}^{\infty}$ is a sequence of fuzzy mappings on X satisfying the following condition:*
there exist $a_1, a_2, a_3 \in (0,1)$ with $a_1 + a_2 + a_3 < 1$ such that for any $i, j \in \mathbb{N}$, $x, y \in X$,

$$H((T_ix)_{\alpha(x)}, (T_jy)_{\alpha(y)}) \leq a_1d(x,y)+a_2d(x,(T_ix)_{\alpha(x)})+a_3d(y,(T_jy)_{\alpha(y)})$$

Then there exists an $x_* \in X$ such that $(\bigcap_{i=1}^{\infty} T_ix_*)_{(x_*)} \geq \alpha(x_*)$.
In addition, if $\alpha(x_*) = 1$ then x_* is a common fixed point of $\{T_i\}_{i=1}^{\infty}$.

remark 3.10. Let $F : X \to CB(X)$. We define a fuzzy mapping $T : X \to CBF(X)$ by $Tx = \chi_{Fx}$.

Note that for $x \in X$, $x \in Fx$ if and only if $\{x\} \subset Tx$.

If we have a sequence $\{F_i\}$ of multivalued mappings from X into $CB(X)$ instead of $\{T_i\}$ in Theorem 3.1 and Theorem 3.6, then the conclusions hold.

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