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CONFORMAL CHANGE OF THE CONNECTION FOR THE FIRST CLASS IN 5–DIMENSIONAL $^*g^{\lambda\nu}$–UNIFIED FIELD THEORY

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I. INTRODUCTION.

The conformal change in a generalized 4-dimensional Riemannian space connected by an Einstein's connection was primarily studied by HLAVATÝ([6], 1957), CHUNG ([4], 1968) also investigated the same topic in 4-dimensional $^*g$-unified field theory was also studied by CHUNG([2], 1988).

The Einstein's connection induced by the conformal change for all classes in 3-dimensional case and for the second and third classes in 5-dimensional case investigated by CHO([1], 1992).

In the present paper, we investigate change of Einstein's connection induced by the conformal change in 5-dimensional $^*g$-unified field theory. These topics will be studied for the first class in 5-dimensional case.

II. PRELIMINARIES.

This chapter is a brief collection of basic concepts, notations, theorems, and results needed in our further considerations. They may be

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referred to CHUNG([3], 1982; [2], 1988), CHO([1], 1992).

2.1. n-dimensional $^*g$-unified field theory.

The n-dimensional $^*g$-unified field theory (n-$^*g$-UFT hereafter) was originally suggested by HLAVATÝ([6], 1957) and systematically introduced by CHUNG([5], 1963).

Let X_n[1] be an n-dimensional generalized Riemannian manifold, referred to a real coordinate system x^ν obeying coordinate transformations $x^\nu \rightarrow x^{\nu'}$, for which

$$\mathrm{Det}\left(\left(\frac{\partial x}{\partial x'}\right)\right) \neq 0. \tag{2.1}$$

In the usual Einstein's n-dimensional unified field theory (n-g-UFT hereafter), the manifold X_n is endowed with a general real nonsymmetric tensor $g_{\lambda\mu}$ which may be split into its symmetric part $h_{\lambda\mu}$ and skew-symmetric part $k_{\lambda\mu}$[2] :

$$g_{\lambda\mu} = h_{\lambda\mu} + k_{\lambda\mu} \tag{2.2}$$

where

$$\mathrm{Det}((g_{\lambda\mu})) \neq 0, \qquad \mathrm{Det}((h_{\lambda\mu})) \neq 0. \tag{2.3}$$

However, the algebraic structure in our n-$^*g$-UFT is imposed on X_n by the basic real tensor $^*g^{\lambda\nu}$ defined by

$$g_{\lambda\mu}{}^*g^{\lambda\nu} = \delta^\nu_\mu. \tag{2.4}$$

It may also be split into a symmetric part $^*h^{\lambda\nu}$ and a skew-symmetric part $^*k^{\lambda\nu}$:

$$^*g^{\lambda\nu} = {}^*h^{\lambda\nu} + {}^*k^{\lambda\nu} \tag{2.5a}$$

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[1] Throughout the present paper, we assumed that $n \geq 2$.

[2] Throughout this paper, Greek indices are used for holonomic components of tensors. In X_n all indices take the values $1, \cdots, n$ and follow the summation convention.

where

$$(2.5)b \qquad \mathrm{Det}((^*g^{\lambda\nu})) \neq 0, \quad \mathrm{Det}((^*h^{\lambda\nu})) \neq 0.$$

In virtue of the second relation of (2.5)b, we may define a unique tensor $^*h_{\lambda\mu}$ by

$$(2.6) \qquad {}^*h_{\lambda\mu}{}^*h^{\lambda\nu} = \delta^\nu_\mu.$$

In our n-$^*h$-UFT, we use both $^*h_{\lambda\mu}$ and $^*h^{\lambda\nu}$ as the tensors for raising and/or lowering indices of the tensors defined in X_n in the usual manner, with the exception of tensors $g_{\lambda\mu}$, $h_{\lambda\mu}$, and $k_{\lambda\mu}$ in order to avoid the notational confusions.

The space X_n is connected by a general real connection $\Gamma^\nu_{\omega\mu}$ with the following transformation rule :

$$(2.7) \qquad \Gamma^{\nu'}_{\omega'\mu'} = \frac{\partial x^{\nu'}}{\partial x^\alpha}\left(\frac{\partial x^\beta}{\partial x^{\omega'}} \cdot \frac{\partial x^\gamma}{\partial x^{\mu'}}\Gamma^\alpha_{\beta\gamma} + \frac{\partial^2 x^\alpha}{\partial x^{\omega'}\partial x^{\mu'}}\right)$$

and satisfies the system of equations

$$(2.8)a \qquad D_\omega{}^*g^{\lambda\nu} = -2S_{\omega\alpha}{}^{\nu}{}^*g^{\lambda\alpha}$$

where D_ω denotes the covariant derivative with respect to $\Gamma^\nu_{\lambda\mu}$ and

$$(2.9) \qquad S_{\lambda\mu}{}^\nu = \Gamma^\nu_{[\lambda\mu]}$$

is the torsion tensor of $\Gamma^\nu_{\lambda\mu}$. In virtue of (2.4), the system (2.8) is equivalent to the system of original Einstein's equations

$$(2.8)b \qquad D_\omega g_{\lambda\mu} = 2S_{\omega\mu}{}^\alpha g_{\lambda\alpha}.$$

REMARK (2.1). When the system (2.8) admits a unique solution, the connection $\Gamma^\nu_{\lambda\mu}$ will be represented in terms of

$$\begin{cases} \text{the tensor } {}^*g_{\lambda\nu} \text{ in } n\text{-}{}^*g\text{-UFT} \\ \text{the tensor } g_{\lambda\mu} \text{ in } n\text{-}g\text{-UFT} \end{cases}.$$

In our further considerations, the following scalars, tensors, abbreviations, and notations for $p = 0, 1, 2, \cdots$ are frequently used :

$$(2.10)a \qquad \begin{array}{l} {}^*\mathfrak{g} = \mathrm{Det}(({}^*g_{\lambda\mu})) \neq 0, \ \ {}^*\mathfrak{h} = \mathrm{Det}(({}^*h_{\lambda\mu})) \neq 0, \\ {}^*\mathfrak{k} = \mathrm{Det}(({}^*k_{\lambda\mu})) \neq 0, \end{array}$$

$$(2.10)b \qquad {}^*g = \frac{{}^*\mathfrak{g}}{{}^*\mathfrak{h}}, \quad {}^*k = \frac{{}^*\mathfrak{k}}{{}^*\mathfrak{h}},$$

$$(2.10)c \qquad K_p = {}^*k_{[\alpha_1}{}^{\alpha_1} \cdots {}^*k_{\alpha_p]}{}^{\alpha_p},$$

$$(2.10)d \qquad {}^{(0)*}k_\lambda{}^\nu = \delta_\lambda^\nu, \ {}^{(1)*}k_\lambda{}^\nu = {}^*k_\lambda{}^\nu, \ {}^{(p)*}k_\lambda{}^\nu = {}^{(p-1)*}k_\lambda{}^{\alpha *}k_\alpha{}^\nu,$$

$$(2.10)e \qquad K_{\omega\mu\nu} = \nabla_\nu{}^*k_{\omega\mu} + \nabla_\omega{}^*k_{\nu\mu} + \nabla_\mu{}^*k_{\omega\nu},$$

$$(2.10)f \qquad \sigma = \begin{cases} 1 & \text{if } n \text{ is odd} \\ 0 & \text{if } n \text{ is even} \end{cases}.$$

Here ∇_ω is the symbolic vector of the covariant derivative with respect to the Christoffel symbols ${}^*\{{}^{\ \nu}_{\lambda\mu}\}$ defined by ${}^*h_{\lambda\mu}$ in the usual way. The scalars and vectors introduced in (2.10) satisfy

$$(2.11)a \qquad K_0 = 1; \ K_n = {}^*k \text{ if } n \text{ is even}; \ K_p = 0 \text{ if } p \text{ is odd},$$

$$(2.11)b \qquad g = 1 + K_2 + \cdots + K_{n-\sigma},$$

$$(2.11)c \qquad {}^{(p)*}k_{\lambda\mu} = (-1)^{p(p)*}k_{\mu\lambda}, \quad {}^{(p)*}k^{\lambda\nu} = (-1)^{p(p)*}k^{\nu\lambda}.$$

Furthermore, we also use the following useful abbreviations, denoting an arbitrary tensor $T_{\omega\mu\nu}$, skew-symmetric in the first two indices, by T :

$$(2.12)a \qquad \overset{pqr}{T} = \overset{pqr}{T}{}_{\omega\mu\nu} = T_{\alpha\beta\gamma}{}^{(p)*}k_\omega{}^{\alpha(q)*}k_\mu{}^{\beta(r)*}k_\nu{}^\gamma,$$

$$(2.12)b \qquad T = T_{\omega\mu\nu} = \overset{000}{T},$$

$$(2.12)c \qquad 2\overset{pqr}{T}_{\omega[\lambda\mu]} = \overset{pqr}{T}_{\omega\lambda\mu} - \overset{pqr}{T}_{\omega\mu\lambda},$$

$$(2.12)d \qquad 2\overset{(pq)r}{T}_{\omega\lambda\mu} = \overset{pqr}{T}_{\omega\lambda\mu} + \overset{qpr}{T}_{\omega\lambda\mu}.$$

We then have

$$(2.13) \qquad \overset{pqr}{T}_{\omega\lambda\mu} = -\overset{qpr}{T}_{\lambda\omega\mu}.$$

If the system (2.8)a admits $\Gamma^{\nu}_{\lambda\mu}$, using the above abbreviations it was shown that the connection is of the form

$$(2.14) \qquad \Gamma^{\nu}_{\omega\mu} = {}^*\{{}^{\nu}_{\omega\mu}\} + S_{\omega\mu}{}^{\nu} + U^{\nu}{}_{\omega\mu}$$

where

$$(2.15) \qquad U_{\nu\omega\mu} = \overset{100}{S}_{(\omega\mu)\nu} + \overset{(10)0}{S}_{\nu(\omega\mu)}.$$

The above two relations show that *our problem of determining* $\Gamma^{\nu}_{\omega\mu}$ *in terms of* ${}^*g^{\lambda\nu}$ *is reduced to that of studying the tensor* $S_{\omega\mu}{}^{\nu}$. On the other hand, it has also been shown that the tensor $S_{\omega\mu}{}^{\nu}$ satisfies

$$(2.16) \qquad S = B - 3\overset{(110)}{S}$$

where

$$(2.17) \qquad 2B_{\omega\mu\nu} = K_{\omega\mu\nu} + 3K_{\alpha[\mu\beta}{}^*k_{\omega]}{}^{\alpha *}k_{\nu}{}^{\beta}.$$

2.2. Some results for the first class in 5-$^*g$-UFT.

In this section, we introduce some results 5-$^*g$-UFT without proof, which are needed in our subsequent considerations.

DEFINITION (2.2). In n-$^*g$-UFT, the tensor $^*g_{\lambda\mu}(^*k_{\lambda\mu})$ is said to be :

(1) of the first class if $K_{n-\sigma} \neq 0$
(2) of the second class with jth category ($j \geq 1$) if

$$K_{2j} \neq 0, \quad K_{2j+2} = K_{2j+4} = \cdots = K_{n-\sigma} = 0$$

(3) of the third class if

$$K_2 = K_4 = \cdots = K_{n-\sigma} = 0.$$

Therefore, in 5-$^*g$-UFT we have three cases; namely, the first class when $K_2 \neq 0$, $K_4 \neq 0$, the second class when $K_2 \neq 0$, $K_4 = 0$, and the third class when $K_2 = K_4 = 0$. In this case the relation (2.11)b is reduced to

THEOREM (2.3). *(For the first class) The following recurrence relations hold in 5-$^*g$-UFT :*

$$(2.18) \quad {}^{(p+5)*}k_\lambda{}^\nu + K_2{}^{(p+3)*}k_\lambda{}^\nu + K_4{}^{(p+1)*}k_\lambda{}^\nu = 0. \quad (p = 0, 1, \cdots)$$

THEOREM (2.4). *(For the first class in 5-$^*g$-UFT). A necessary and sufficient condition for the existence and uniqueness of the solution of (2.8)a is*

$$(2.19) \qquad gAB(C^2 - 4K_4D^2) \neq 0$$

where

$$(2.20) \qquad \begin{aligned} A &= 1 - K_2 + K_4, & B &= 1 - K_4, \\ C &= 1 - K_2 + 5K_4, & D &= K_2 - 2. \end{aligned}$$

If the condition (2.19) is satisfied, the unique solution of (2.16) is given by

$$\begin{aligned} gAB(C^2 - 4K_4D^2)(S-B) &= (1-K_2+5K_4)\underset{1}{B} \\ &\quad + 2[1-2K_2+(K_2)^2-5K_4]\underset{2}{B} + 2(K_2-2)\underset{3}{B} \end{aligned} \tag{2.21}$$

where

$$\begin{aligned} \underset{1}{B} = &- \overset{222}{B} + 2(\overset{(20)2}{B} + K_4\overset{(20)4}{B} - \overset{(10)1}{B} + \overset{112}{B}) \\ &+ (K_2-1)[\overset{220}{B} + K_4\overset{114}{B} + 3K_2(K_4)^2\overset{224}{B}] \\ &+ K_4[2\overset{220}{B} - K_4B - \overset{330}{B} + 2(K_2)^2\overset{(31)4}{B} + K_2K_4\overset{(40)2}{B} \\ &+ 4\overset{(10)3}{B} + K_2(\overset{112}{B} + 2\overset{(10)3}{B} - 4K_4\overset{(10)1}{B}) \\ &+ [(K_2)^2 - 1 + K_4](2\overset{(10)1}{B} - \overset{110}{B} + K_4\overset{(20)4}{B} \\ &+ (K_2 - 1 + 2K_4)[\overset{220}{B} + K_4\overset{332}{B} + 2(K_2)^2K_4\overset{330}{B}] \end{aligned} \tag{2.22a}$$

$$\begin{aligned} \underset{2}{B} =&2[(K_4)^2B + \overset{(12)3}{B} + K_4\overset{(10)3}{B} + (K_4)^2\overset{(30)1}{B} - K_4\overset{(20)2}{B}] \\ &+ K_2\overset{112}{B} + (1+K_2)[2K_4\overset{(10)1}{B} + (K_4)^2\overset{334}{B} + 3K_4\overset{(40)4}{B}] \\ &- \overset{222}{B} - K_4[\overset{220}{B} + K_2\overset{(31)0}{B} + 2(K_2)^2\overset{(13)2}{B} + K_2K_4\overset{(30)3}{B}] \\ &+ (1+K_2)(1+3K_4)(2\overset{(12)1}{B} - \overset{110}{B}) \end{aligned} \tag{2.22b}$$

$$\begin{aligned} \underset{3}{B} =&(K_4)^2B + 2\overset{(12)3}{B} + [2K_4 - (K_2)^2]\overset{112}{B} \\ &+ K_2[\overset{220}{B} + (K_4)^2\overset{(31)0}{B}] \\ &+ K_4[K_2\overset{220}{B} - 2\overset{(12)1}{B} + 2\overset{(20)2}{B} - (K_2)^2\overset{(30)3}{B} + K_2K_4\overset{(32)3}{B}] \\ &+ K_4[2 + 2K_2 - (K_2)^2](\overset{110}{B} + K_2\overset{330}{B} - K_4\overset{334}{B}) \\ &- K_4(1+K_2)[2\overset{(10)3}{B} - \overset{220}{B} - K_2\overset{(10)1}{B} + (K_2)^2\overset{(23)1}{B}]. \end{aligned} \tag{2.22c}$$

III. Conformal change of the 5-dimensional Einstein's connection for the first class.

In this final chapter we investigate the change $\Gamma^{\nu}_{\lambda\mu} \to \overline{\Gamma}^{\nu}_{\lambda\mu}$ of the connection induced by the conformal change of the tensor ${}^*g^{\lambda\nu}$, using the recurrence relations and theorems introduced in the preceding chapter.

Consider two manifolds X_n ($\overline{X}_n$), on which the differential geometric structure is imposed by a general real tensor ${}^*g^{\lambda\nu}$ ($\overline{{}^*g}^{\lambda\nu}$) through the connection $\Gamma^{\nu}_{\lambda\mu}$ ($\overline{\Gamma}^{\nu}_{\lambda\mu}$) given by (2.14) ((3.1)) :

$$\overline{\Gamma}^{\nu}_{\lambda\mu} = \overline{{}^*\{{}^{\nu}_{\lambda\mu}\}} + \overline{S}_{\lambda_\mu}{}^{\nu} + \overline{U}^{\nu}_{\lambda\mu}. \tag{3.1}$$

We say that X_n and $\overline{X}_n$ are conformal if and only if

$$\overline{{}^*g}^{\lambda\nu}(x) = e^{-\Omega}{}^*g^{\lambda\nu}(x) \tag{3.2}$$

where $\Omega = \Omega(x)$ is an at least twice differentiable function. This conformal change enforces a change of the connection $\Gamma^{\nu}_{\lambda\mu}$. An explicit representation of the change of 5-dimensional connection $\Gamma^{\nu}_{\lambda\mu}$ for the first class will be exhibited in this chapter.

Agreement (3.1). Throughout this section, we agree that, if T is a function of ${}^*g^{\lambda\nu}$, then we denote $\overline{T}$ the same function of $\overline{{}^*g}^{\lambda\nu}$. In particular, if T is a tensor, so is $\overline{T}$. Furthermore, the indices of T ($\overline{T}$) will be raised and/or lowered by means of ${}^*h^{\lambda\nu}$ ($\overline{{}^*h}^{\lambda\nu}$) and/or ${}^*h_{\lambda\mu}$ ($\overline{{}^*h}_{\lambda\mu}$).

The results in the following theorems needed in our further considerations.

They may be referred to CHO([1], 1992).

Theorem (3.2). *In n-${}^*g$-UFT, the conformal change (3.2) induces the following changes :*

$$\begin{aligned} {}^{(p)}\overline{{}^*k}_{\lambda\mu} &= e^{\Omega(p)}{}^*k_{\lambda\mu}, \quad {}^{(p)}\overline{{}^*k}_{\lambda}{}^{\nu} = {}^{(p)}{}^*k_{\lambda}{}^{\nu}, \\ {}^{(p)}\overline{{}^*k}^{\lambda\nu} &= e^{-\Omega(p)}{}^*k^{\lambda\nu} \end{aligned} \tag{3.3a}$$

$$\overline{{}^*g} = {}^*g, \quad \overline{K_p} = K_p, \qquad (p = 1, 2, \cdots). \tag{3.3b}$$

THEOREM (3.3). *In n-$^*g$-UFT, the conformal change (3.2) induces the following changes :*

$$\overline{^*\{{}^{\;\nu}_{\omega\mu}\}} = {}^*\{{}^{\;\nu}_{\omega\mu}\} + 2\delta^{\nu}_{(\omega}\Omega_{\mu)} - \frac{1}{2}{}^*h_{\omega\mu}{}^*h^{\nu\alpha}\Omega_\alpha \tag{3.4}$$

where

$$\Omega_\mu = \partial_\mu \Omega.$$

THEOREM (3.4). *(For the First class in 5-$^*g$-UFT). The change of the tensor $B_{\omega\mu\nu}$ induced by the conformal change (3.2) may be given by*

$$\begin{aligned} \overline{B}_{\omega\mu\nu} =& e^{\Omega}(B_{\omega\mu\nu} + {}^*k_{\nu[\omega}\Omega_{\mu]} - {}^*k_{\omega\mu}\Omega_\nu \\ & - {}^*h_{\nu[\omega}{}^*k_{\mu]}{}^{\delta}\Omega_\delta + 2{}^{(2)*}k_{\nu[\omega}{}^*k_{\mu]}{}^{\delta}\Omega_\delta + {}^*k_{\omega\mu}{}^{(2)*}k_\nu{}^\delta\Omega_\delta). \end{aligned} \tag{3.5}$$

Now, we are ready to derive representations of the changes $S_{\omega\mu}{}^{\nu} \to \overline{S}_{\omega\mu}{}^{\nu}$ in 5-$^*g$-UFT for the first class induced by the conformal change (3.2).

THEOREM (3.5). *The conformal change (3.2) induces the following change :*

$$\begin{aligned} \overline{\overset{pqq}{B}}_{\omega\mu\nu} =& e^{\Omega}[\overset{ppq}{B}_{\omega\mu\nu} + (-1)^p\{2{}^{(p+q+2)*}k_{\nu[\omega}{}^{(p+1)*}k_{\mu]}{}^{\delta} \\ & + {}^{(2p+1)*}k_{\omega\mu}{}^{(2+q)*}k_\nu{}^\delta - {}^{(2p+1)*}k_{\omega\mu}{}^{(q)*}k_\nu{}^\delta \\ & + {}^{(p+q+1)*}k_{\nu[\omega}{}^{(p)*}k_{\mu]}{}^{\delta} - {}^{(p+q)*}k_{\nu[\omega}{}^{(p+1)*}k_{\mu]}{}^{\delta}\}\Omega_\delta]. \\ & \begin{pmatrix} p &=& 0,1,2,3,4,\cdots \\ q &=& 0,1,2,3,4,\cdots \end{pmatrix} \end{aligned} \tag{3.6}$$

Proof. In virtue of (3.5), (2.12)a, (2.10)d, (3.3)a, the relation (3.6) may be derived as follows :

$$\begin{aligned} \overline{\overset{pqq}{B}}_{\omega\mu\nu} &= \overline{\overset{000}{B}}_{\alpha\beta\gamma}{}^{(p)*}\overline{k}_{\omega}{}^{\alpha(p)*}\overline{k}_{\mu}{}^{\beta(q)*}\overline{k}_{\nu}{}^{\gamma} \\ &= e^{\Omega}(B_{\alpha\beta\gamma} + {}^*k_{\gamma[\alpha}\Omega_{\beta]} - {}^*k_{\alpha\beta}\Omega_\gamma - {}^*h_{\gamma[\alpha}{}^*k_{\beta]}{}^{\delta}\Omega_\delta \\ &\quad + 2{}^{(2)*}k_{\gamma[\alpha}{}^*k_{\beta]}{}^{\delta}\Omega_\delta + {}^*k_{\alpha\beta}{}^{(2)*}k_\gamma{}^\delta\Omega_\delta)^{(p)*}k_\omega{}^{\alpha\,(p)*}k_\mu{}^{\beta\,(q)*}k_\nu{}^\gamma \\ &= (\text{the right-hand side of (3.6)}). \end{aligned}$$

REMARK (3.6). From the above relation (3.6), we can be obtained the tensors. $\overset{110}{B}$, $\overset{112}{B}$, $\overset{114}{B}$, $\overset{220}{B}$, $\overset{222}{B}$, $\overset{224}{B}$, $\overset{330}{B}$, $\overset{332}{B}$, $\overset{334}{B}$, induced by the conformal change.

THEOREM (3.7). *The conformal change (3.2) induces the following changes :*

$$(3.7)a \quad 2\overline{\overset{(10)1}{B}}_{\omega\mu\nu} = e^{\Omega}[2\overset{(10)1}{B}_{\omega\mu\nu} + (-2^{(4)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta} + 2^{(2)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta} - {}^{*}k_{\nu[\omega}{}^{(2)*}k_{\mu]}{}^{\delta})\Omega_{\delta} - {}^{(3)*}k_{\nu[\omega}\Omega_{\mu]}],$$

$$(3.7)b \quad \begin{aligned} 2\overline{\overset{(20)2}{B}}_{\omega\mu\nu} = e^{\Omega}[2\overset{(20)2}{B}_{\omega\mu\nu} &+ (2^{(6)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta} \\ &+ 2^{(4)*}k_{\nu[\omega}{}^{(3)*}k_{\mu]}{}^{\delta} + 2^{(3)*}k_{\omega\mu}{}^{(4)*}k_{\nu}{}^{\delta} \\ &- 2^{(3)*}k_{\omega\mu}{}^{(2)*}k_{\nu}{}^{\delta} + {}^{(3)*}k_{\nu[\omega}{}^{(2)*}k_{\mu]}{}^{\delta} \\ &- {}^{(2)*}k_{\nu[\omega}{}^{(3)*}k_{\mu]}{}^{\delta} - {}^{(4)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta})\Omega_{\delta} \\ &+ {}^{(5)*}k_{\nu[\omega}\Omega_{\mu]}], \end{aligned}$$

$$(3.7)c \quad \begin{aligned} 2\overline{\overset{(40)2}{B}}_{\omega\mu\nu} = e^{\Omega}[2\overset{(40)2}{B}_{\omega\mu\nu} &+ (2^{(8)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta} \\ &+ 2^{(4)*}k_{\nu[\omega}{}^{(5)*}k_{\mu]}{}^{\delta} + {}^{(5)*}k_{\omega\mu}{}^{(4)*}k_{\nu}{}^{\delta} \\ &+ {}^{(3)*}k_{\nu[\omega}{}^{(4)*}k_{\mu]}^{\delta} - 2^{(5)*}k_{\omega\nu}{}^{(2)*}k_{\nu}{}^{\delta} \\ &- {}^{(6)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta} - {}^{(2)*}k_{\nu[\omega}{}^{(5)*}k_{\mu]}{}^{\delta})\Omega_{\delta} \\ &+ {}^{(7)*}k_{\nu[\omega}\Omega_{\mu]}], \end{aligned}$$

$$(3.7)d \quad \begin{aligned} 2\overline{\overset{(31)4}{B}}_{\omega\mu\nu} = e^{\Omega}[2\overset{(31)4}{B}_{\omega\mu\nu} &+ (-2^{(9)*}k_{\nu[\omega}{}^{(2)*}k_{\mu]}{}^{\delta} \\ &- 2^{(7)*}k_{\nu[\omega}{}^{(4)*}k_{\mu]}{}^{\delta} - 2^{(5)*}k_{\omega\mu}{}^{(6)*}k_{\nu}{}^{\delta} \\ &+ {}^{(7)*}k_{\nu[\omega}{}^{(2)*}k_{\mu]}{}^{\delta} - {}^{(6)*}k_{\nu[\omega}{}^{(3)*}k_{\mu]}{}^{\delta} \\ &+ 2\,{}^{(5)*}k_{\omega\mu}{}^{(4)*}k_{\nu}{}^{\delta} - {}^{(8)*}k_{\nu[\omega}{}^{*}k_{\mu]}{}^{\delta})\Omega_{\delta}]. \end{aligned}$$

Proof. The first relation (3.7)a follows by substituting (2.12), (3.3)a, (3.5) into

$$
\begin{aligned}
2 \overset{\overline{(10)1}}{B}{}_{\omega\mu\nu} &= \overset{\overline{101}}{B}{}_{\omega\mu\nu} + \overset{\overline{011}}{B}{}_{\omega\mu\nu} \\
&= \overset{\overline{000}}{B}{}_{\alpha\mu\beta}\overline{{}^*k}_{\omega}{}^{\alpha}\overline{{}^*k}_{\nu}{}^{\beta} + \overset{\overline{000}}{B}{}_{\omega\alpha\beta}\overline{{}^*k}_{u}{}^{\alpha}\overline{{}^*k}_{v}{}^{\beta}
\end{aligned}
$$

we have (3.7)a.

The remaining relations may be proved similarly. □

THEOREM (3.8). *The change $S_{\omega\mu}{}^{\nu} \to \overline{S}_{\omega\mu}{}^{\nu}$ induced by conformal change (3.2) may be represented by*

$$
\begin{aligned}
\overline{S}_{\omega\mu}{}^{\nu} = S_{\omega\mu}{}^{\nu} + E[& a_1 {}^*k_{\omega\mu}\Omega^{\nu} + a_2 {}^*k_{[\omega}{}^{\nu}\Omega_{\mu]} \\
&+ a_3 {}^*k_{[\omega}{}^{\nu(2)*}k_{\nu]}{}^{\delta}\Omega_{\delta} + a_4 {}^*k_{\omega\mu}{}^{(2)*}k_{v}{}^{\delta}\Omega_{\delta} \\
&+ a_5 {}^*k_{[\omega}{}^{\nu(4)*}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_6 {}^*k_{\omega\mu}{}^{(4)*}k_{\nu}{}^{\delta}\Omega_{\delta} \\
&+ a_7{}^{(2)*}k_{[\omega}{}^{\nu*}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_8{}^{(2)*}k_{[\omega}{}^{\nu(3)*}k_{\mu]}{}^{\delta}\Omega_{\delta} \\
&+ a_9{}^{(3)*}k_{\omega\mu}\Omega^{\nu} + a_{10}{}^{(3)*}k_{[\omega}{}^{\nu}\Omega_{\mu]} \\
&+ a_{11}{}^{(3)*}k_{[\omega}{}^{\nu(2)*}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_{12}{}^{(3)*}k_{\omega\mu}{}^{(2)*}k^{\nu\delta}\Omega_{\delta} \\
&+ a_{13}{}^{(3)*}k_{[\omega}{}^{\nu(4)*}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_{14}{}^{(3)*}k_{\omega\mu}{}^{(4)*}k^{\nu\delta}\Omega_{\delta} \\
&+ a_{15}{}^{(4)*}k_{[\omega}{}^{\nu*}k_{\mu]}{}^{\delta}\Omega_{\delta} + a_{16}{}^{(4)*}k_{[\omega}{}^{\nu(3)*}k_{\mu]}{}^{\delta}\Omega_{\delta} \\
&+ a_{17} {}^*k_{[\omega}{}^{\nu*}k_{\mu]}{}^{\delta}\Omega_{\delta}]
\end{aligned}
\tag{3.8}
$$

where

$$
\begin{aligned}
a_1 =& -5\alpha^4 + \alpha^3(30 + 3\beta + 11\beta^2 - 5\beta^4 + 10\beta^5) \\
&+ \alpha^2(-6 + 13\beta - 14\beta^2 + 7\beta^4 - 7\beta^5 + 2\beta^6) \\
&+ \alpha(-3\beta + \beta^2) - \frac{1}{E}, \\
a_2 =& -\frac{5}{2}\alpha^4(10 + 5\beta + \beta^2) + \alpha^3(-30 - 4\beta + 15\beta^2 + 18\beta^3 + \beta^4) \\
&+ \frac{\alpha^2}{2}(16 - 39\beta + 5\beta^2 + 3\beta^3 + 7\beta^4 - 6\beta^5) \\
&+ \alpha(-1 + \beta^2) + \frac{1}{E}, \\
a_3 =& -\frac{5}{2}\alpha^4(1 + 10\beta^2 - 6\beta^3) + \frac{\alpha^3}{2}(124 + 143\beta - 3\beta^2 - 16\beta^3 + 14\beta^4) \\
&+ \frac{\alpha^2}{2}(-113 + 119\beta + 33\beta^2 - 19\beta^3 - 12\beta^4 + 4\beta^5) \\
&+ \alpha(-4 + 9\beta + 11\beta^2 - 11\beta^3) - \beta + 1, \\
a_4 =& 5\alpha^5(-5\beta + \beta^2) + \alpha^4(-30 - 8\beta - 2\beta^2 - 31\beta^3 + 6\beta^4) \\
&+ \alpha^3(-49 + 3\beta + 16\beta^2 + 14\beta^3 - 6\beta^4) \\
&+ \alpha^2(11 - 11\beta + 11\beta^2 - 13\beta^3 - 4\beta^4 + 12\beta^5 - 4\beta^6) \\
&+ \alpha(-2 + 3\beta) + \beta(1 - \beta) + \frac{1}{E}, \\
a_5 =& 20\alpha^5(1 + \beta) + 2\alpha^4(-7 - 23\beta + 7\beta^2 - 12\beta^3) \\
&+ \alpha^3(26 + 35\beta - 52\beta^2 - 18\beta^3 - 2\beta^4 + 4\beta^5) \\
&+ \alpha^2(-2 + 16\beta + 21\beta^2 - 9\beta^3 - 2\beta^4) + 2\alpha(-\beta + 2\beta^2), \\
a_6 =& -5\alpha^4(6\beta + \beta^2 + 2\beta^3) + \alpha^3(-30 - 24\beta - 11\beta^2 - 11\beta^3 \\
&- 6\beta^4 + 4\beta^5) + \alpha^2(1 - 6\beta - 4\beta^2 + 4\beta^4) + \alpha(3 - 5\beta + 2\beta^2), \\
a_7 =& \frac{-5\alpha^4}{2}(31 + 22\beta + 2\beta^2) + \frac{\alpha^3}{2}(-112 - 15\beta + 97\beta^2 + 76\beta^3 + 4\beta^4) \\
&+ \frac{\alpha^2}{2}(-19 - 169\beta - 5\beta^2 + 25\beta^3 + 6\beta^4 - 12\beta^5) \\
&+ \alpha(-5 + 11\beta + 33\beta^2 - 14\beta^3) \\
&+ \alpha(1 - \alpha - 7\alpha^2 - 4\alpha^3) - 1 + \frac{2}{E},
\end{aligned}
$$

$$
\begin{aligned}
a_8 =& 5\alpha^4(1-\beta-\beta^2+6\beta^3)+\frac{\alpha^3}{2}(-38-74\beta-53\beta^2+34\beta^3-20\beta^4)\\
&+2\alpha^2(200-56\beta-11\beta^2-5\beta^3+8\beta^4)+\alpha(15-21\beta+7\beta^3)\\
&+\beta(3-6\beta+2\beta^2)+3,\\
a_9 =& -20\alpha^4\beta^2+\alpha^3(5+6\beta^2-6\beta^3)\\
&+\alpha^2(26+67\beta-13\beta^2-10\beta^3+6\beta^4+2\beta^5)\\
&+\alpha(1+\beta+9\beta^2-17\beta^3-4\beta^4+8\beta^5-12\beta^6)-\beta+3,\\
a_{10} =& \frac{5}{2}\alpha^4(-1+\beta)+\frac{\alpha^3}{2}(-6-62\beta-75\beta^2-7\beta^3)\\
&+\frac{\alpha^2}{2}(9-47\beta+\beta^2+11\beta^2+41\beta^4+2\beta^5)\\
&+\frac{\alpha}{2}(14+32\beta-37\beta^2-9\beta^3+2\beta^4+8\beta^5-6\beta^6)\\
&+\beta(-3-\beta+2\beta^2)+2,\\
a_{11} =& 5\alpha^4(-1+3\beta-3\beta^2)+2\alpha^3(-42+25\beta+8\beta^2-64\beta^3+30\beta^4)\\
&+\frac{\alpha^2}{2}(-8+64\beta+127\beta^2-53\beta^3-30\beta^4+26\beta^5)\\
&+\frac{\alpha}{2}(-58-39\beta+21\beta^2+53\beta^3-23\beta^4-10\beta^5+4\beta^6)\\
&+\beta(2-\beta)-5,\\
a_{12} =& 15\alpha^5+\alpha^4(-19-3\beta+5\beta^2+\beta^3)\\
&+\alpha^3(-2-33\beta-9\beta^2-16\beta^3-33\beta^4+2\beta^5+10\beta^6-4\beta^7)\\
&\frac{\beta}{2}(2-18\alpha+11\alpha^2)+1,\\
a_{13} =& 10\alpha^4(3+3\beta+4\beta^2)+\frac{\alpha^3}{2}(-72-91\beta-36\beta^2+100\beta^3-72\beta^4)\\
&+\frac{\alpha^2}{2}(-6-23\beta+73\beta^2+24\beta^3-32\beta^5+16\beta^6)\\
&+\alpha(-1-11\beta-16\beta^2+3\beta^3+10\beta^4+20\beta^5+8\beta^6),
\end{aligned}
$$

$$
\begin{aligned}
a_{14} =& 5\alpha^4(2+4\beta-\beta^2+3\beta^3)+\alpha^3(-8-39\beta-65\beta^2-18\beta^3+2\beta^4)\\
&+\alpha^2(10-19\beta-73\beta^2-55\beta^3-7\beta^4-9\beta^5+4\beta^6)\\
&+\alpha(-6+17\beta-7\beta^2-9\beta^3-7\beta^4+12\beta^5)+2\beta^2(-2+\beta),\\
a_{15} =& -5\alpha^4-\frac{\alpha^3}{2}(92+283\beta+163\beta^2+14\beta^3)\\
&+\frac{\alpha^2}{2}(-46-4\beta+99\beta^3+88\beta^4+4\beta^5)\\
&+\frac{\alpha}{2}(58+43\beta-49\beta^2-7\beta^3+19\beta^4+6\beta^5-12\beta^6)\\
&+\beta(\beta-4)+3,\\
a_{16} =& 10\alpha^4(-1+2\beta-3\beta^2)+\frac{\alpha^3}{2}(25+38\beta-100\beta^2-10\beta^3+60\beta^4)\\
&+\frac{\alpha^2}{2}(284+71\beta-103\beta^2-76\beta^3+34\beta^4-20\beta^5)\\
&+\frac{\alpha}{2}(-81+69\beta+47\beta^2-65\beta^3+50\beta^4)+\beta(4\beta^2-6\beta+7)-5,\\
a_{17} =& -2\alpha^2-\frac{1}{E},
\end{aligned}
$$

where

$$\alpha = K_4, \quad \beta = K_2, \quad E = \frac{C^2-4\alpha D^2}{gAB}.$$

Proof. In virtue of (2.21) and Agreement (3.1), we have

$$
\begin{aligned}
\overline{gAB}(\overline{C}^2-4\overline{K}_4\overline{D}^2)(\overline{S}-\overline{B}) &= (1-\overline{K}_2+5\overline{K}_4)\underset{1}{\overline{B}}\\
&+2[1-2\overline{K}_2+(\overline{K}_2)^2-5\overline{K}_4]\underset{2}{\overline{B}}+2(\overline{K}_2-2)\underset{3}{\overline{B}}
\end{aligned}
\tag{3.9}
$$

The relation (3.8) follows by substituiting (3.3)a, (3.3)b, (2.18), (3.5), (2.12)a, (3.6), (3.7) into (3.9). □

Now the change of the tensor $U^\nu{}_{\omega\mu}$ can be obtained by using relation (2.15), (3.8) and Agreement (3.1).

Therefore, the change of the connection $\Gamma^\nu_{\omega\mu}$ can be calculated using the following equation

$$\overline{\Gamma}^\nu_{\omega\mu} = \overline{{}^*\{{}^{\,\nu}_{\omega\mu}\}}+\overline{S}_{\omega\mu}{}^\nu+\overline{U}^\nu_{\omega\mu} \tag{3.10}$$

by only substituiting for $\overline{^*\{{}^{\nu}_{\omega\mu}\}}$, $\overline{S}$ and $\overline{U}$ into (3.10).

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