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ON S—IDENTIFICATION MAPS

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1. Introduction.

Let X and Y be topological spaces on which no separation axioms are assumed unless explicitly stated. A subclass $\tau^* \subseteq \wp(X)$ is called a supra-topology(Mashhour et al. 1983) on X if $X \in \tau^*$ and τ^* is closed under arbitrary union. (X, τ^*) is called a supra-topological space (or supra-space). The members of τ^* are called supra-open sets. And let(X, τ) be a topological spaces and τ^* be a supra-topology on X. We call τ^* a supra-topology associated with τ if $\tau \subseteq \tau^*$. Let (X, τ) and (Y,τ_1) be topological spaces and let τ^* be an associated supra-topology with τ. A function $f: X \to Y$ is an S-continuous function if the inverse image of each open set in Y is τ^*-supra-open in X. And let τ_1^* be an associated supra-topology with τ_1. A function $f: (X, \tau^*) \to (Y, \tau_1^*)$ is S^*-continuous if the inverse image of each τ_1^*-supra-open set is τ^*-supra-open. Let f be S^*-continuous and g is S-continuous, then gof is S-continuous. But the inverse may not be true. In this paper, we get the following property: Let $f: X \to Y$ be an S-identification map and $g: Y \to Z$ be a map. Then g is S(S^*)-continuous if and only if gof is S(S^*)-continuous. Now we assume that τ^* is the fixed associated supra-topology with τ.

2. S-identification map.

2.1. DEFINITION.. Let (X, τ) be a topological space and τ^* be an associated supra-topology with τ. Let Y be an arbitrary set and $p: X \to Y$ be surjection. The identification supra-topology in Y determined by p is

$$\tau^*(p) = \{U \subseteq Y \mid p^{-1}(U) \text{ is } \tau^* - \text{supra-open in } X\}.$$

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Then $\tau^*(p)$ is a supra-topology on Y, and B is supra-closed in $(Y, \tau^*(p))$ if and only if $p^{-1}(B)$ is τ^*-supra-closed in X.

2.2. THEOREM. *$\tau^*(p)$ is the largest supra-topology in (Y, τ_1) for which $p: X \to Y$ is an S^*-continuous function.*

Proof. If τ_2^* is any other associated supra-topology with τ_1 and $p: (X, \tau^*) \to (Y, \tau_2^*)$, then for U in τ_2^*, $p^{-1}(U)$ is a supra-open in X. Thus $U \in \tau^*(p)$.

2.3. DEFINITION.. Let (X, τ_1) and (Y, τ_2) be two spaces and τ_1^* and τ_2^* be two associated supra-topologies with τ_1, τ_2, respectively. An S^*-continuous surjection $p: X \to Y$ is an S-identification map (or S-identification) whenever the associated supra-topology τ_2^* with τ_2 is exactly $\tau^*(p)$.

By the definition of S-identification, the S^*-continuous identity map $id: (X, \tau_1) \to (X, \tau_2)$ is an S-identification if and only if $\tau_1^* = \tau_2^*$. Although $\tau_1 \neq \tau_2$, id may be an S-identification.

Example. Let $X = \{a, b, c\}$. Let $\tau_1 = \{\emptyset, \{b\}, \{a, b\}, X\}$ and $\tau_2 = \{\emptyset, \{b\}, \{b, c\}, X\}$. Consider $\tau_1^* = \tau_1 \cup \{\{b, c\}, \{a, c\}\}$ and $\tau_2^* = \tau_2 \cup \{\{a, b\}, \{a, c\}\}$. Although $\tau_1 \neq \tau_2$, id is an S-identification, since $\tau_1^* = \tau_2^*$.

2.4. THEOREM. *If $p: (X, \tau^*) \to (Y, \tau_2^*)$ is an S^*-continuous S^*-open(S^*-closed) surjection, then p is an S-identification.*

Proof. Since p is S^*-continuous, $\tau_2^* \subseteq \tau^*(p)$. Let $U \in \tau^*(p)$. Then $p^{-1}(U)$ is supra-open in X, and since p is S^*-open and surjective, $U = p(p^{-1}(U))$ is supra-open in τ_2^*. Therefore $\tau_2^* = \tau^*(p)$.

2.5. THEOREM. *let $p: (X, \tau_1^*) \to (Y, \tau_2^*)$ be S^*-continuous. If there is an S^*-continuous map $q: (Y, \tau_2^*) \to (X, \tau_1^*)$ such that $poq = id$, then p is an S-identification.*

Proof. Let $U \in \tau^*(p)$. Then $p^{-1}(U)$ is supra-open. $q^{-1}(p^{-1}(U)) = U$ is supra-open in (Y, τ_1), and so p is an S-identification.

Recall that A is p-saturated if and only if $A = p^{-1}p(A)$, and the p-load of any $A \subseteq X$ is the p-saturated set $p^{-1}p(A) \supseteq A$.

2.6. THEOREM. *Let* $p: (X, \tau_1^*) \to (Y, \tau_2^*)$ *be an S-identification Then* p *is* S^**-open if and only if the* p*-load of each supra-open in* X *is also supra-open in* X.

Proof. Let U be supra-open in X. If p is S^*-open, then $p(\mathrm{U})$ is supra-open in Y, Since p is S^*-continuous, $p^{-1}(p(U))$ is supra-open in X. For the converse, let U be supra-open in X. Since $p^{-1}p(U)$ is supra-open in X, we obtain that p(U) is supra-open in τ_2^*.

2.7. THEOREM. *Let* $p: (X, \tau_1^*) \to (Y, \tau_2^*)$ *be an S-identification. Then* p *is S-open if and only if the* p*-load of each open in* X *is supra-open in* X.

Proof. Obvious.

Remark. Let $f: X \to Y$ and $g: Y \to Z$ be two functions and f be S^*-continuous. If g is S-continuous, then $gof: X \to Z$ is S-continuous. In general, the converse is not true. But if f is an S-identification, we obtain the following property.

2.8. THEOREM. *if* $p: (X, \tau_1^*) \to (Y, \tau_2^*)$ *is an S-identification and* $g: (Y, \tau_2^*) \to (Z, \tau_3^*)$ *is a function. Then* g *is* $S(S^*)$*-continuous if and only if* gof *is* $S(S^*)$*-continuous.*

Proof. Since p is S^*-continuous, gop is $S(S^*)$-continuous. For the converse, assume that (gop) is $S(S^*)$-continuous and let U be any open(supra-open) in Z. Then $(gop)^{-1}(U) = p^{-1}(g^{-1}(U))$ is supra-open in X. Since p is S-identification, $g^{-1}(U)$ is supra-open in (Y, τ_2^*). Thus g is $S(S^*)$-continuous.

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