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A STUDY ON THE ALGEBRA OF P–VECTORS IN A GENERALIZED 2–DIMENSIONAL RIEMANNIAN MANIFOLD X_2

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1. INTRODUCTION

Let X_2 be two-dimensional Riemannian manifold referred to a real coordinate system x^ν, which obeys only coordinate transformations $x^\nu \to \overline{x}^\nu$, for which

$$(1.1)\qquad Det\left(\left(\frac{\partial \overline{x}}{\partial x}\right)\right) \neq 0$$

and is endowed with a real nonsymmetric tensor $g_{\lambda\mu}$ which may be split into its symmetric part $h_{\lambda\mu}$ and skew-symmetric part $k_{\lambda\mu}$:

$$(1.2)\qquad g_{\lambda\mu} = h_{\lambda\mu} + k_{\lambda\mu}$$

where

$$(1.3)\qquad \mathfrak{g} = Det((g_{\lambda\mu})) \neq 0, \quad \mathfrak{h} = Det((h_{\lambda\mu})) < 0, \quad \mathfrak{t} = Det((k_{\lambda\mu}))$$

We may define a unique tensor $h^{\lambda\nu}$ by

$$(1.4)\qquad h_{\lambda\mu}h^{\lambda\nu} = \delta_\mu^\nu$$

which together with $h_{\lambda\mu}$ will serve for rasing and/or lowering indices of tensors in X_2 in the usual manner.

In our subsequent considerations, the following scalars and tensors are frequently used;

$$(1.5)\qquad g = \frac{\mathfrak{g}}{\mathfrak{h}} \quad , \quad k = \frac{\mathfrak{t}}{\mathfrak{h}}$$

$$(1.6)\qquad \mathfrak{g} = \mathfrak{h} + \mathfrak{t},$$

$$(1.7)\qquad {}^{(0)}k_\lambda^\nu = \delta_\lambda^\nu, \quad {}^{(p)}k_\lambda^\nu = {}^{(p-1)}k_\lambda^\alpha k_\alpha^\nu \qquad (p = 1, 2, \cdots)$$

$$(1.8)\qquad \mathfrak{t} = \Omega^2 > 0, \quad k = \frac{\Omega^2}{\mathfrak{h}} < 0 \quad , \text{ where } \Omega = k_{12}$$

$$(1.9)\qquad Det(({}^{(2)}k_{\lambda\mu})) = \frac{\Omega^4}{\mathfrak{h}} < 0 \qquad , {}^{(2)}k_\alpha^\alpha = -\frac{2\Omega^2}{\mathfrak{h}} > 0$$

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(1.10) $k^{12} = \dfrac{\Omega}{\mathfrak{h}}$

(1.11) $k_1^2 = \dfrac{h_{11}\Omega}{\mathfrak{h}}$, $\quad k_2^1 = -\dfrac{h_{22}\Omega}{\mathfrak{h}}$, $\quad k_2^2 = -k_1^1 = \dfrac{h_{12}\Omega}{\mathfrak{h}}$

(1.12) ${}^{(2)}k_1^1 = {}^{(2)}k_2^2 = -\dfrac{\Omega^2}{\mathfrak{h}}$, $\quad {}^{(2)}k_1^2 = {}^{(2)}k_2^1 = 0$

(1.13) ${}^{(2)}k_{11} = -\frac{h_{11}\Omega^2}{\mathfrak{h}}, {}^{(2)}k_{22} = -\dfrac{h_{22}\Omega^2}{\mathfrak{h}}, {}^{(2)}k_{12} = {}^{(2)}k_{21} = -\dfrac{h_{12}\Omega^2}{\mathfrak{h}}$

Furthermore, we use $E^{\alpha_1\alpha_2\cdots\alpha_n}$ ($e_{\alpha_1\alpha_2\cdots\alpha_n}$) as the contravariant (covariant) indicator of weight $1(-1)$.

The eigenvalue M and the corresponding eigenvector a^ν in X_2, defined by

(1.14) $$(Mh_{\nu\lambda} - k_{\nu\lambda})a^\nu = 0 \quad (M: \text{ a sclar })$$

are called basic scalars and basic vectors of X_2 , respectively.

There are exactly two linearly independent basic vectors a^ν satisfying (1.14) , where the corresponding basic scalars M are given by

(1.15) $$\underset{1}{M} = -\underset{2}{M} = \sqrt{-K}$$

It is well-known that the basic vectors $\underset{1}{a}^\nu$ and $\underset{2}{a}^\nu$ are null-vectors and not perpendicular.

2. Some algebra of ${}^{(3)}k_{\lambda\mu}$ in X_2

Theorem 1. *In X_2 ,*

(2.1) $$Det(({}^{(3)}k_{\lambda\mu})) = k^3\mathfrak{h}$$

Proof.

$$\begin{aligned} {}^{(3)}k_{\lambda\mu} &= {}^{(2)}k_\lambda^\alpha k_{\alpha\mu} \\ &= k_\lambda^\beta k_\beta^\alpha k_{\alpha\mu} \\ &= h^{\beta a}k_{a\lambda}h^{\alpha b}k_{\beta b}k_{\alpha\mu} \end{aligned}$$

Hence,

$$Det(({}^{(3)}k_{\lambda\mu})) = \frac{\mathfrak{t}^3}{\mathfrak{h}^2} = \frac{\Omega^6}{\mathfrak{h}^2} = k^3\mathfrak{h} > 0.$$

THEOREM 2. *In X_2, the components of tensors may be given by*

(2.2)a $\quad {}^{(3)}k_1^1 = -{}^{(3)}k_2^2 = \dfrac{h_{12}\Omega^3}{\mathfrak{h}^2}$

(2.2)b $\quad {}^{(3)}k_1^2 = -\dfrac{h_{11}\Omega^3}{\mathfrak{h}^2} \quad , \quad {}^{(3)}k_2^1 = \dfrac{h_{22}\Omega^3}{\mathfrak{h}^2}$

(2.2)c $\quad {}^{(3)}k_{11} = {}^{(3)}k_{22} = 0$

(2.2)d $\quad {}^{(3)}k_{21} = -{}^{(3)}k_{12} = \dfrac{\Omega^3}{\mathfrak{h}}$

Proof.

$$\begin{aligned}
{}^{(3)}k_1^1 &= {}^{(2)}k_1^\alpha k_\alpha^1 = {}^{(2)}k_1^1 k_1^1 + {}^{(2)}k_1^2 k_2^1 \\
&= (-\frac{\Omega^2}{\mathfrak{h}})(-\frac{h_{12}\Omega}{\mathfrak{h}}) + 0 = \frac{h_{12}\Omega^3}{\mathfrak{h}^2}
\end{aligned}$$

$$\begin{aligned}
{}^{(3)}k_1^2 &= {}^{(2)}k_1^\alpha k_\alpha^2 = {}^{(2)}k_1^1 k_1^2 + {}^{(2)}k_1^2 k_2^2 \\
&= (-\frac{\Omega^2}{\mathfrak{h}})(\frac{h_{11}\Omega}{\mathfrak{h}}) + 0 = -\frac{h_{11}\Omega^3}{\mathfrak{h}^2}
\end{aligned}$$

$${}^{(3)}k_{11} = {}^{(2)}k_1^\alpha k_{\alpha 1} = {}^{(2)}k_1^2 k_{21} = 0$$

$$\begin{aligned}
{}^{(3)}k_{12} &= {}^{(2)}k_1^\alpha k_{\alpha 2} = {}^{(2)}k_1^1 k_{12} \\
&= -(\frac{\Omega^2}{\mathfrak{h}})\Omega = -\frac{\Omega^3}{\mathfrak{h}}
\end{aligned}$$

REMARK 1.

(2.3) $Det(({}^{(3)}k_{\lambda\mu})) = k^3\mathfrak{h}$ in X_2

We note that

$$Det(({}^{(3)}k_{\lambda\mu})) = \begin{vmatrix} {}^{(3)}k_{11} & {}^{(3)}k_{12} \\ {}^{(3)}k_{21} & {}^{(3)}k_{22} \end{vmatrix} = \begin{vmatrix} 0 & -\frac{\Omega^3}{\mathfrak{h}} \\ \frac{\Omega^3}{\mathfrak{h}} & 0 \end{vmatrix} = \frac{\Omega^6}{\mathfrak{h}^2} = k^3\mathfrak{h}$$

REMARK 2. In X_2,

$${}^{(3)}k_{\lambda\mu} = -kk_{\lambda\mu}$$

From the fact that $k_{11} = k_{22} = 0$, we have

$${}^{(3)}k_{12} = -\frac{\Omega^3}{\mathfrak{h}} = (-\frac{\Omega^2}{\mathfrak{h}})\Omega = -k\Omega = -kk_{12}$$

DEFINITION 1. The eigenvalue $\overline{M}$ and the corresponding eigenvector $\overline{A}^{\nu}$ in X_2 defined by

$$(\overline{M}h_{\nu\lambda} - {}^{(3)}k_{\nu\lambda})\overline{A}^{\nu} = 0 \ (\overline{M} : \text{a scalar})$$

are called 3-scalars and 3-vectors, respectively.

THEOREM 3. *In X_2, there are exactly two linearly independent 3-sclars $\overline{M}$ given by*

$$\underset{1}{\overline{M}} = -\underset{2}{\overline{M}} = \sqrt{-k^3}$$

Proof.

$$\begin{aligned} E^{\omega\mu}E^{\alpha\beta}h_{\omega\alpha}\,{}^{(3)}k_{\mu\beta} &= E^{\omega\mu}E_{\omega\beta}\,{}^{(3)}k_{\mu}^{\beta} = \mathfrak{h}\ E^{\omega\mu}e_{\omega\beta}\,{}^{(3)}k_{\mu}^{\beta} \\ &= \mathfrak{h}\ \delta_{\beta}^{\mu}\,{}^{(3)}k_{\mu}^{\beta} = \mathfrak{h}\,{}^{(3)}k_{\beta}^{\beta} = \mathfrak{h}\,(\frac{\Omega^3}{\mathfrak{h}^2}h_{12} - \frac{\Omega^3}{\mathfrak{h}^2}h_{12}) \\ &= 0 \end{aligned}$$

Now,

$$\begin{aligned} &2\ Det((\overline{M}h_{\nu\lambda} - {}^{(3)}k_{\nu\lambda})) \\ &= E^{\omega\mu}E^{\alpha\beta}\,(\overline{M}h_{\omega\alpha} - {}^{(3)}k_{\omega\alpha})\,(\overline{M}h_{\mu\beta} - {}^{(3)}k_{\mu\beta}) \\ &= 2\,\overline{M}^2\ \mathfrak{h} - 2\,\overline{M}E^{\omega\mu}E^{\alpha\beta}h_{\omega\alpha}\,{}^{(3)}k_{\mu\beta} + 2Det((\,{}^{(3)}k_{\lambda\mu})) \\ &= 2\,\overline{M}^2\ \mathfrak{h} + 2k^3\mathfrak{h} = 2\mathfrak{h}\,(\overline{M}^2 + k^3) = 0 \end{aligned}$$

Therefore,

$$\overline{M} = \pm\sqrt{-k^3}$$

THEOREM 4. *The basic vector of $\underset{1}{a}^{\nu}$ and $\underset{2}{a}^{\nu}$ of X_2 are also 3-vectors of X_2.*

Proof. ${}^{(3)}k_{\nu\lambda}\,\underset{i}{a}^{\nu} = {}^{(2)}k_{\nu}^{\alpha}\ k_{\alpha\lambda}\,\underset{i}{a}^{\nu} = k_{\nu}^{\beta}\ k_{\beta}^{\alpha}k_{\alpha\lambda}\,\underset{i}{a}^{\nu} = \underset{i}{M}\underset{i}{a}^{\beta}k_{\beta}^{\alpha}\ k_{\alpha\lambda}$
$= \underset{i}{M}^2 k_{\alpha\lambda}a^{\alpha} = \underset{i}{M}^3 h_{\nu\lambda}\underset{i}{a}^{\nu}$

Therefore $\underset{i}{a}^{\nu}$ is the 3-vector with 3-sclar $\overline{M}$ given by

$$\overline{M} = \underset{i}{M}^3 = \pm(\sqrt{-k})^3 \qquad (i = 1, 2)$$

3. Some algebra of ${}^{(p)}k_{\lambda\mu}$ in X_2

Theorem 5. *In X_2, we have*

$$(3.1)\text{a} \quad {}^{(p)}k_{\lambda\mu} = (-k)^{\frac{p}{2}}\, h_{\lambda\mu} \qquad (\,p:\ even\,)$$

$$(3.1)\text{b} \quad {}^{(p)}k_{\lambda\mu} = (-k)^{\frac{p-1}{2}}\, k_{\lambda\mu} \qquad (\,p:\ odd\,)$$

Proof. By induction on p, the theorem may be proved.

Theorem 6. $Det(({}^{(p)}k_{\lambda\mu})) = k^p \mathfrak{h}$ *in* X_2

Proof.

(case 1) p is even

$$Det(({}^{(p)}k_{\lambda\mu})) = \begin{vmatrix} {}^{(p)}k_{11} & {}^{(p)}k_{12} \\ {}^{(p)}k_{21} & {}^{(p)}k_{22} \end{vmatrix} = \begin{vmatrix} (-k)^{\frac{p}{2}}\, h_{11} & (-k)^{\frac{p}{2}}\, h_{12} \\ (-k)^{\frac{p}{2}}\, h_{21} & (-k)^{\frac{p}{2}}\, h_{22} \end{vmatrix} = k^p \mathfrak{h}$$

(case 2) p is odd

$$\begin{aligned} Det(({}^{(p)}k_{\lambda\mu})) &= \begin{vmatrix} (-k)^{\frac{p-1}{2}}\, k_{11} & (-k)^{\frac{p-1}{2}}\, k_{12} \\ (-k)^{\frac{p-1}{2}}\, k_{21} & (-k)^{\frac{p-1}{2}}\, k_{22} \end{vmatrix} \\ &= (-k)^{p-1}\mathfrak{t} \;=\; k^{p-1}\Omega^2 \;=\; k^{p-1}(k\mathfrak{h}) \;=\; k^p\mathfrak{h} \end{aligned}$$

Remark 3. Another proof of Theorem (3.2) may be obtained as in the following.

By induction on p,

in case of $p = 1$, $\quad Det((k_{\lambda\mu})) = \mathfrak{t} = \Omega^2 = k\mathfrak{h}$

Assume that the theorem is proved for $p-1$.

i.e. $Det(({}^{(p-1)}k_{\lambda\mu})) = k^{p-1}\mathfrak{h}$

Now, using the induction hypothesis

$$Det(({}^{(p)}k_{\lambda\mu})) = Det(({}^{(p-1)}k^{\alpha}_{\lambda}k_{\alpha\mu})) = Det(({}^{(p-1)}k_{\lambda\beta}h^{\alpha\beta}k_{\alpha\mu}))$$

$$= (\,k^{p-1}\mathfrak{h})(\frac{1}{\mathfrak{h}})(\Omega^2) = k^{p-1}\Omega^2 = k^{p-1}\frac{\Omega^2}{\mathfrak{h}}\mathfrak{h} = k^p\mathfrak{h}$$

Hence the theorem is proved for all p.

Theorem 7. *In X_2, the components of tensors may be given by*

$$(3.3)\text{a} \quad {}^{(p)}k^2_1 = {}^{(p)}k^1_2 = 0 \qquad (\,p\ :\ even)$$

$$(3.3)\text{b} \quad {}^{(p)}k^2_1 = (-k)^{\frac{p-1}{2}}\, k^2_1 = (-k)^{\frac{p-1}{2}}\,(\frac{h_{11}\Omega}{\mathfrak{h}})$$

$$\phantom{(3.3)\text{b}} \quad {}^{(p)}k^1_2 = (-k)^{\frac{p-1}{2}}\, k^1_2 = (-k)^{\frac{p-1}{2}}\,(-\frac{h_{22}\Omega}{\mathfrak{h}}) \qquad (\,p: odd)$$

(3.3)c ${}^{(p)}k_1^1 = {}^{(p)}k_2^2 = (-k)^{\frac{p}{2}}$ $\quad$ (p : *even*)

(3.3)d ${}^{(p)}k_1^1 + {}^{(p)}k_2^2 = 0$ $\quad$ (p : *odd*)

Proof. Let p be even.

(a) By induction on p,
in case of $p = 2$, we have ${}^{(2)}k_1^2 = 0$ by (1.12).
Assume that the theorem hold for $p-2$,
i.e. we assume that ${}^{(p-2)}k_1^2 = 0$
Now,

$$
\begin{aligned}
{}^{(p)}k_1^2 &= {}^{(p-1)}k_1^{\alpha}k_{\alpha}^2 \\
&= {}^{(p-2)}k_1^{\beta}k_{\beta}^{\alpha}\ k_{\alpha}^2 \\
&= {}^{(p-2)}k_1^1\ (\ k_1^1\ k_1^2 + k_1^2\ k_2^2\ ) \quad (\because {}^{(p-2)}k_1^2 = 0) \\
&= {}^{(p-2)}k_1^1[(-\frac{h_{12}\Omega}{\mathfrak{h}})(\frac{h_{11}\Omega}{\mathfrak{h}}) + (\frac{h_{11}\Omega}{\mathfrak{h}})(\frac{h_{12}\Omega}{\mathfrak{h}})] \qquad (3.4) \\
&= 0
\end{aligned}
$$

(c) By induction on p,
in case of $p = 2$, we have ${}^{(2)}k_1^1 = -k = (-k)^{\frac{2}{2}}$
Assume that the theorem hold for $p-2$,
i.e. we assume that ${}^{(p-2)}k_1^1 = (-k)^{\frac{p-2}{2}}$
Now,

$$
\begin{aligned}
{}^{(p)}k_1^1 &= {}^{(p-2)}k_1^{\beta}k_{\beta}^{\alpha}k_{\alpha}^1 \\
&= {}^{(p-2)}k_1^1k_1^{\alpha}k_{\alpha}^1 + {}^{(p-2)}k_1^2k_2^{\alpha}k_{\alpha}^1 \\
&= {}^{(p-2)}k_1^1k_1^{\alpha}k_{\alpha}^1 \qquad (\because \ by\ (a)) \\
&= {}^{(p-2)}k_1^1\ [k_1^1k_1^1 + k_1^2\ k_2^1\ ] \qquad (3.5) \\
&= (-k)^{\frac{p-2}{2}}\ [(-\frac{h_{12}\Omega}{\mathfrak{h}})^2 + (\frac{h_{11}\Omega}{\mathfrak{h}})(-\frac{h_{22}\Omega}{\mathfrak{h}})] \\
&= (-k)^{\frac{p-2}{2}}\ [-\frac{(h_{11}h_{22} - (h_{12})^2)\Omega^2}{\mathfrak{h}^2}] \\
&= (-k)^{\frac{p-2}{2}}\ (-\frac{\Omega^2}{\mathfrak{h}}) = (-k)^{\frac{p-2}{2}}\ (-k) = (-k)^{\frac{p}{2}}
\end{aligned}
$$

Hence the theorem holds for all even numbers p.

DEFINITION 2. The eigenvalue H and the corresponding eigenvector P^ν in X_2 defined by

(3.6) $$(Hh_{\nu\lambda} - {}^{(p)}k_{\nu\lambda})\,P^\nu = 0 \qquad (\,H : \text{a scalar})$$

are called p-sclars and p-vectors, respectively.

THEOREM 8. (1) *In X_2, there is exactly one p-scalar H, given by* $H = (-k)^{\frac{p}{2}}$ *(p : even).*
(2) *In X_2, there are exactly two p-scalars H, given by* $H = \pm(-k)^{\frac{p-2}{2}}$ *(p : odd) .*

Proof. (1) Let p be even.

(3.7) $$\begin{aligned} E^{\omega\mu}E^{\alpha\beta}h_{\omega\alpha}\,{}^{(p)}k_{\mu\beta} &= E^{\omega\mu}E_{\omega\beta}\,{}^{(p)}k^{\beta}_{\mu} \\ &= \mathfrak{h}\,E^{\omega\mu}e_{\omega\beta}\,{}^{(p)}k^{\beta}_{\mu} = \mathfrak{h}\,\delta^{\mu}_{\beta}\,{}^{(p)}k^{\beta}_{\mu} \\ &= \mathfrak{h}\,{}^{(p)}k^{\beta}_{\beta} = 2(-k)^{\frac{p}{2}}\,\mathfrak{h} \end{aligned}$$

using (3.3)c.
Now,

(3.8) $$\begin{aligned} &2\,Det((H\,h_{\nu\lambda} - {}^{(p)}k_{\nu\lambda})) \\ &= E^{\omega\mu}E^{\alpha\beta}(H\,h_{\omega\alpha} - {}^{(p)}k_{\omega\alpha})(H\,h_{\mu\beta} - {}^{(p)}k_{\mu\beta}) \\ &= 2H^2\mathfrak{h} - 2H\,E^{\omega\mu}E^{\alpha\beta}h_{\omega\alpha}{}^{(p)}k_{\mu\beta} + 2Det(({}^{(p)}k_{\lambda\mu})) \\ &= 2H^2\mathfrak{h} - 2H(2(-k)^{\frac{p}{2}}\,\mathfrak{h}) + 2k^p\mathfrak{h} \\ &= 2\mathfrak{h}\,(H^2 - 2H(-k)^{\frac{p}{2}} + k^p) \\ &= 2\mathfrak{h}\,(H - (-k)^{\frac{p}{2}})^2 \end{aligned}$$

Since the characteristic equation of (3.6) is

(3.9) $Det((H\,h_{\nu\lambda} - {}^{(p)}k_{\nu\lambda})) = 0.$

Note that $H = (-k)^{\frac{p}{2}}$ is a double root of (3.9).

(2) Let p be odd.

$$E^{\omega\mu}E^{\alpha\beta}h_{\omega\alpha}\,{}^{(p)}k_{\mu\beta} = \mathfrak{h}\,{}^{(p)}k^{\beta}_{\beta} = 0 \qquad (\because\ (3.3)d)$$

Since

$$
\begin{aligned}
& 2\ Det((H\ h_{\nu\lambda} - {}^{(p)}k_{\nu\lambda})) \\
& \quad = E^{\omega\mu}E^{\alpha\beta}(H\ h_{\omega\alpha} - {}^{(p)}k_{\omega\alpha})(H\ h_{\mu\beta} - {}^{(p)}k_{\mu\beta}) \\
& \quad = 2H^2\mathfrak{h} - 2H\ E^{\omega\mu}E^{\alpha\beta}h_{\omega\alpha}{}^{(p)}k_{\mu\beta} + 2Det(({}^{(p)}k_{\lambda\mu})) \\
& \quad = 2H^2\mathfrak{h} - 0 + 2k^p\mathfrak{h} \\
(3.10) \qquad & \quad = 2\mathfrak{h}\,(H^2 + k^p) \\
& \quad = 0
\end{aligned}
$$

The characteristic equation of (3.6) is

(3.11) $\qquad Det((H\ h_{\nu\lambda} - {}^{(p)}k_{\nu\lambda})) = 0,$

so that $H^2 = -k^p$ satisfies (3.11) , that is
$H = \pm\,(-k)^{\frac{p}{2}}$.

THEOREM 9. *The basic vectors* $\underset{i}{a}^\nu$ $(i = 1, 2)$ *of* X_2 *are also p-vectors of* X_2.

Proof. We claim that

(3.12) $\qquad {}^{(p)}k_{\nu\lambda}\underset{i}{a}^\nu = M^p h_{\nu\lambda}\underset{i}{a}^\nu$

Indeed , by induction on p ,

in case of $p = 1$, $\qquad k_{\nu\lambda}\underset{i}{a}^\nu = M h_{\nu\lambda}\underset{i}{a}^\nu \qquad (\because (2.6))$

Suppose that $\qquad {}^{(p-1)}k_{\nu\lambda}\underset{i}{a}^\nu = M^{p-1}\ h_{\nu\lambda}\underset{i}{a}^\nu$

Then ,

$$
\begin{aligned}
{}^{(p)}k_{\nu\lambda}\underset{i}{a}^\nu &= {}^{(p-1)}k_\nu^\alpha k_{\alpha\lambda}\underset{i}{a}^\nu \\
&= M^{p-1}k_{\alpha\lambda}\underset{i}{a}^\nu \qquad \text{(by induction hyphothesis)} \\
&= M^{p-1}(M h_{\nu\lambda}\underset{i}{a}^\nu\ ) \\
&= M^p h_{\nu\lambda}\underset{i}{a}^\nu
\end{aligned}
$$

Therefore , $\underset{i}{a}^\nu$ is the p- vector with p - scalar H given by
$H = M^p = (-k)^{\frac{p}{2}}$

THEOREM 10. *If p is even , then every vector of X_2 is a p-vector of X_2 corresponding to p-sclar $H = (-k)^{\frac{p}{2}}$.*

Proof. (3.1)a gives
${}^{(p)}k_{\nu\lambda} = (-k)^{\frac{p}{2}} h_{\nu\lambda}$ in X_2.
Therefore (3.6) can be written as

$$(H - (-k)^{\frac{p}{2}}) h_{\nu\lambda} P^\nu = 0 \tag{3.13}$$

Since $\mathfrak{h} \neq 0$ and the relation (3.13) holds for every vector P^ν when $H = (-k)^{\frac{p}{2}}$, hence our theorem is proved.

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