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ON M-OPEN MAPPINGS

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Abstract. In this paper, we introduce m-open(closed) mappings by m-sets, and obtain a number of their properties. In particular, m-open(closed) mappings are used to extend known results for α-open mapping, semi-open mappings and preopen mappings.

1. Introduction

Let X, Y and Z be topological spaces on which no separation axioms are assumed unless explicity stated. Let S be a subset of X. The closure (resp. interior, boundary) of S will be denoted by S^- (resp. $S^0, b(S)$). A subset S of X is called semi-open set[1] (resp. preopen set[2], α-set[3]) if $S \subset S^{0-}$ (resp. $S \subset S^{-0}, S \subseteq S^{0-0}$). The complement of a semi-open set (resp. preopen set, α-set) is called semi-closed set(resp. preclosed set, α-closed set). The family of all semi-open sets(resp. preopen sets, α-sets) in X will be denoted by $SO(X)$ (resp. $PO(X)$, $\alpha(X)$). A function $f : X \to Y$ is called semi-open mapping[5] (resp. pre-open mapping[2], α-open mapping[6]) if $f(U) \in SO(X)$ (resp. $f(U) \in PO(X)$, $f(U) \in \alpha(X)$) for each open set U of X.

A subclass $\tau^* \subset P(X)$ is called a supratopology on X if $X \in \tau^*$ and τ^* is closed under arbitrary union. (X, τ^*) is called a supratopological space. The members of τ^* are called supraopen sets[7]. Let (X, τ) be a topological space and τ^* be a supratopology on X. We call τ^* a supratopology associated with τ if $\tau \subset \tau^*$. The topological space (X, τ) with τ^* will be denoted by (X, τ, τ^*). Let (X, τ) be topological space and (Y, μ^*) be supratopological space. A function $f : X \to Y$ is an s-open(s-closed) mapping if the image of each open(closed) set in X is a supraopen(supraclosed) set in Y[7]. Let (X, τ^*) be a supratopological

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space. A subset A of X is called an m-set with τ^* if $A \cap T \in \tau^*$ for all $T \in \tau^*$[4]. The class of all m-sets with τ^* will be denoted by τ_m. A subset B of X is called an m-closed set if the complement of B is an m-set. In this paper, we introduce m-open(closed) mappings by m-sets, and obtained a number of their properties. In particular, a mapping $f : (X, \tau) \to (Y, \mu^*)$ is m-open if and only if for each $x \in X$ and each open set U of X containing x, there exists an m-open set $W \subset Y$ containing $f(x)$ such that $W \subset f(U)$. And m-open(closed) mappings are used to extend known results for α-open mapping, semi-open mappings and preopen mappings. Finally we get that if $f : (X, \tau) \to (Y, \mu, PO(Y))$ is α-open, then f is m-open.

m-open(closed) mappings

DEFINITION 2.1. Let (X, τ) be a topological space and (Y, μ^*) be a supratopological space. A mapping $f : (X, \tau) \to (Y, \mu^*)$ is called an m-open(m-closed) mapping if the image of each open(closed) set in X is an m-set(m-closed set).

From the above definition, m-open(m-closed) mappings are s-open (s-closed) mappings. The converse of these implications is not true as the following example illustrates.

EXAMPLE 2.2. Let $X = \{a, b, c\}$ and $\tau = \{\phi, X, \{a, b\}\}$. Consider $\tau^* = \{\phi, X, \{a, b\}, \{b, c\}, \{c, a\}\}$. Then $\tau_m = \{\phi, X\}$. If $f : X \to X$ is the identity mapping, then f is an s-open mapping but it is not an m-open mapping.

By the bellow two examples, we know the independence of the concepts of open mappings and m-open mappings.

EXAMPLE 2.3. Let $X = \{a, b, c, d\}$, $\tau = \{\phi, X, \{a, b\}\}$ and $\tau^* = \{\phi, X, \{a, b\}, \{b, d\}, \{a, b, d\}\}$. Then $\tau_m = \{\phi, X, \{a, b, d\}\}$. If $f : X \to X$ is the identity mapping, then f is an open mapping but it is not an m-open mapping.

EXAMPLE 2.4. Let $X = \{a, b, c, d\}$ and $\tau = \{\phi, X, \{a, b\}\}$. Let $Y = \{1, 2, 3, 4\}$, $\mu = \{\phi, Y, \{1\}\}$ and

$$\mu^* = \{\phi, Y, \{1\}, \{2, 3\}, \{1, 2, 3\}, \{2, 3, 4\}, \{1, 4\}\}.$$

Then $\mu_m = \{\phi, Y, \{1\}, \{2,3\}, \{1,2,3\}\}$. If a mapping $g : X \to Y$ is defined by $g(a) = 2$, $g(b) = 3$, $g(c) = 1$, and $g(d) = 1$, then g is an m-open mapping but it is not an open mapping.

Now we get the following thing, by the definitions m-sets and m-open mappings.

THEOREM 2.5. *If $f : (X,\tau) \to (Y,\mu,\mu^*)$ is an open mapping and $\mu \subset \mu_m$, then f is an m-open mapping and s-open mapping.*

COROLLARY 2.6. *If $f : (X,\tau) \to (Y,\mu,SO(Y))$ is an open mapping, then f is an α-open mapping and semi-open mapping.*

Proof. Since every m-set with the supratopology $SO(Y)$ is an α-set, $\mu \subset \alpha(X) = \mu_m$. By Theorem 2.5, we get that f is α-open and semi-open. □

LEMMA 2.7. *For a topological space $(Y,\mu,PO(Y))$, we have $\alpha(Y) \subset \mu_m$.*

Proof. Let $A \in \alpha(Y)$. Then for all $B \in \mu^* = PO(Y)$, $A \cap B \subset A^{0-0} \cap B^{-0} \subset (A \cap B)^{-0}$. Thus $A \cap B \subset PO(Y)$. □

THEOREM 2.8. *If $f : (X,\tau) \to (Y,\mu,PO(Y))$ is α-open, then f is m-open and pre-open.*

Proof. By Lemma 2.7, it is obvious. □

THEOREM 2.9. *A mapping $f : (X,\tau) \to (Y,\mu^*)$ is m-open if and only if for each $x \in X$ and each open set U of X containing x, there exists an m-open set $W \subset Y$ containing $f(x)$ such that $W \subset f(U)$.*

Proof. Suppose that f is an m-open mapping. For each $x \in X$ and each open set U of X containing x, $f(U)$ is an m-open set in Y containing $f(x)$. Set $W = f(U)$, then W is an m-open set containing $f(x)$ such that $W \subset f(U)$.

Conversely, it is obvious. □

COROLLARY 2.10. *Let $f : (X,\tau) \to (Y,\mu,SO(Y))$ be a mapping, then f is α-open if and only if for each $x \in X$ and each open set U of X containing x, there exists an α-open set $W \subset Y$ containing $f(x)$ such that $W \subset f(U)$.*

Proof. It follows from $\alpha(Y) = \mu_m$ in the supratopology $SO(Y)$. □

THEOREM 2.11. *A mapping $f : (X, \tau) \to (Y, \mu^*)$ is an m-closed mapping if and only if $mcl(f(A)) \subset f(A^-)$ for each $A \subset X$.*

Proof. Suppose that f is an m-closed mapping. For each $A \subset X$, since $f(A^-)$ is an m-closed set, we have $f(A^-) = mcl(f(A^-)) \supset mcl f(A)$.

Conversely, let A closed in X. Since $mcl(f(A)) \subset f(A^-) = f(A)$, $f(A)$ is an m-closed set, and hence f is m-closed. □

COROLLARY 2.12. *Let $f : (X, \tau) \to (Y, \mu, SO(Y))$ be a mapping, then f is an α-closed mapping if and only if $cl_\alpha(f(A)) \subset f(A^-)$ for each $A \subset X$.*

THEOREM 2.13. *A mapping $f : (X, \tau) \to (Y, \mu^*)$ is m-open (m-closed) if and only if a mapping $f : (X, \tau) \to (Y, \mu_m)$ is open (closed).*

Proof. If $f : (X, \tau) \to (Y, \mu^*)$ is an m-open(m-closed) mapping then image of each open(closed) set in X is an m-set(m-closed set) in μ^*. Since m-sets(m-closed sets) in μ^* are open(closed) sets in μ_m.

The converse is obvious. □

Theorem 2 of [8] follows immediately from Theorem 2.13 and $\mu_m = \alpha(X)$. Thus we get the following thing.

COROLLARY 2.14. *A mapping $f : (X, \tau) \to (Y, \mu, SO(Y))$ is α-open (α-closed) if and only if $f : (X, \tau) \to (Y, \mu_m)$ is an open(closed) mapping.*

THEOREM 2.15. *A mapping $f : (X, \tau) \to (Y, \mu^*)$ is an m-open mapping. If $W \subset Y$ and $F \subset X$ is a closed set containing $f^{-1}(W)$, then there exists an m-closed set $H \subset Y$ containing W such that $f^{-1}(H) \subset F$.*

Proof. Let $W \subset Y$ and let $F \subset X$ be a closed set containing $f^{-1}(W)$. Set $H = Y - f(X - F)$. Then H is an m-closed set, $f^{-1}(H) \subset F$, and $W \subset H$. □

Corollary 2.16. *Let $f : (X, \tau) \to (Y, \mu, SO(Y))$ be a mapping, then if f is α-open, $W \subset Y$, and $F \subset X$ is a closed set containing $f^{-1}(W)$, then there exists an α-closed set, $H \subset Y$ containing W such that $f^{-1}(H) \subset F$.*

Remarks. Let $f : (X, \tau) \to (Y, \mu, \mu^*)$ be a function. In conclusion we can get the following diagrams :

(1) In $\mu \subset \mu_m$, open $\Longrightarrow$ m-open $\Longrightarrow$ s-open

(2) In $\mu^* = SO(Y)$, open $\Longrightarrow$ m-open(=α-open) $\Longrightarrow$ semi-open

(3) In $\mu^* = PO(Y)$, open $\Longrightarrow$ α-open $\Longrightarrow$ m-open $\Longrightarrow$ pre-open

(4) If f is an open mapping and $g : (Y, \mu) \to (Z, \nu, \nu^*)$ is an m-open mapping then $g \circ f$ is an m-open mapping.

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