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L_p FOURIER-FEYNMAN TRANSFORMS AND CONVOLUTION

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Abstract. Let $\mathcal{F}(B)$ be the Fresnel class on an abstract Wiener space (B, H, ω) which consists of functionals F of the form :

$$F(x) = \int_H \exp\{i(h, x)^\sim\}\, df(h), \quad x \in B,$$

where $(\cdot,\cdot)^\sim$ is a stochastic inner product between H and B, and f is in $\mathcal{M}(H)$, the space of all complex-valued countably additive Borel measures on H.

We introduce the concepts of an L_p analytic Fourier-Feynman transform $(1 \le p \le 2)$ and a convolution product on $\mathcal{F}(B)$ and verify the existence of the L_p analytic Fourier-Feynman transforms for functionls in $\mathcal{F}(B)$. Moreover, we verify that the Fresnel class $\mathcal{F}(B)$ is closed under the L_p analytic Fourier-Feynman transform and the convolution product, respectively. And we investigate some interesting properties for the n-repeated L_p analytic Fourier-Feynman transform on $\mathcal{F}(B)$. Finally, we show that several results in [9] come from our results in Section 3.

1. Introduction

In [2], Brue investigated initially the theory of an L_1 analytic Fourier-Feynman transform on a classical Wiener space, and in [3] Cameron and Storvick introduced the concept of an L_2 analytic Fourier-Feynman transform on a classical Wiener space. In [10], Johnson and Skoug developed an L_p analytic Fourier-Feynman transform theory for $1 \le p \le 2$ which extended the results in [2;3]. In [8;9], Huffman, Park and Skoug

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developed an L_p analytic Fourier-Feynman transform theory on certain classes of functionals defined on a classical Wiener space and they defined a convolution product of two functionals on the classical Wiener space and then showed that the Fourier-Feynman transform of the convolution product was a product of Fourier-Feynman transforms. In [1], the author investigated the L_1 analytic Fourier-Feynman transform theory on the Fresnel class $\mathcal{F}(B)$ of an abstract Wiener space.

The paper is organized as follows. In Section 2, we introduce the basic concepts and the notations for our research. In Section 3, we obtain our main results for the n-repeated L_p analytic Fourier-Feynman transform theory on the Fresnel class $\mathcal{F}(B)$ of an abstract Wiener space. In the last section, we show that several results in [9] come from our results in Section 3.

2. Definitions and Preliminaries

Let H be a real separable infinite dimensional Hilbert space with norm $|\cdot| = \sqrt{\langle\cdot,\cdot\rangle}$, where $\langle\cdot,\cdot\rangle$ is an inner product on H. Let $\|\cdot\|_o$ be a fixed measurable norm on H (for definition see [13]). Let B be the completion of H with respect to the measurable norm $\|\cdot\|_o$ and $\mu_t(\, t>0\,)$ the Gauss measure on H with variance t. Then μ_t induces a cylinder set measure $\tilde{\mu}_t$ on B which in turn extends to a countably additive measure ω_t on $(B, \mathcal{B}(B))$, where $\mathcal{B}(B)$ is the Borel σ-algebra of Borel sets in B. ω_t is called the *Wiener measure* with variance t and it has the following properties:

$$\tag{2.1} \begin{cases} \omega_{st}(E) = \omega_t(s^{-1/2}E), \quad s>0, \\ \omega_t(-E) = \omega_t(E). \end{cases}$$

From now on, we shall use ω instead of ω_1, identifying ω with ω_1.

Let $\{e_n\}$ denote a complete orthonormal set of H such that e_n's are in B^*, the topological dual space of B. For each $h \in H$ and $x \in B$, we define a *stochastic inner product* $(\cdot,\cdot)^\sim$ between H and B as follows:

$$\tag{2.2} (h,x)^\sim = \begin{cases} \lim\limits_{n\to\infty} \sum\limits_{k=1}^{n} \langle h, e_k\rangle (e_k, x) & \text{, if the limit exists} \\ 0 & \text{, otherwise,} \end{cases}$$

where $(\cdot,\cdot)$ is the natural dual pairing between B^* and B.

It is well known [11,12] that for every $h \in H, (h,x)^\sim$ exists for ω_t-a.e. $x \in B$, and $(h,\cdot)^\sim$ is a Borel measurable functional on B having a

Gaussian distribution with mean zero and variance $t|h|^2$ with respect to ω_t. Furthermore, it is obvious that for each real number $\alpha, (\alpha h, x)^\sim = \alpha(h, x)^\sim = (h, \alpha x)^\sim$ holds for every $h \in H$ and $x \in B$.

Let (B, H, ω_t) be an abstract Wiener space. For each $\lambda > 0$, let $\mathcal{S}_\lambda(B)$ be the completion of $\mathcal{B}(B)$ with respect to ω_λ, and let $\mathcal{N}_\lambda(B) = \{A \in \mathcal{S}_\lambda(B) : \omega_\lambda(A) = 0\}$. Let $\mathcal{S}(B) = \bigcap_{\lambda>0} \mathcal{S}_\lambda(B)$, and $\mathcal{N}(B) = \bigcap_{\lambda>0} \mathcal{N}_\lambda(B)$. Every set in $\mathcal{S}(B)$ (or $\mathcal{N}(B)$) is called a *scale-invariant measurable* (or *scale-invariant null*) set. A real (or complex)-valued functional F on B is called *scale-invariant measurable* if F is measurable with respect to $\mathcal{S}(B)$. A property that holds except on a scale-invariant null set is said to hold *scale-invariant almost everywhere* (briefly, s-a.e.). If two functionals F and G are equal s-a.e., then we write $F \approx G$. It is easy to show that this relation $\approx$ is an equivalence relation on the class of functionals on B. For a functional F on B, we will denote by $[F]$ the equivalence class of functionals which are equal to F s-a.e..

Definition 2.1. Let (B, H, ω) be an abstract Wiener space and $\mathcal{M}(H)$ the space of all complex-valued countably additive Borel measures on H. Consider the functional F defined for s-a.e. $x \in B$ by the formula

$$F(x) = \int_H \exp\{i(h, x)^\sim\}\, df(h), \tag{2.3}$$

where f is in $\mathcal{M}(H)$. Let $\mathcal{F}(B)$ denote the collection of equivalence classes $[F]$ of functionals which are equal to F s-a.e. on B. Then we call $\mathcal{F}(B)$ the *Fresnel class* on the abstract Wiener space (B, H, ω).

Remark 2.2. (1) As is customary, we will identify a functional with its equivalence class and think of $\mathcal{F}(B)$ as a class of functionals on B rather than as a class of equivalence classes.

(2) $\mathcal{M}(H)$ is a Banach algebra over the complex fields under the total variation norm $\|\cdot\|$, where the convolution is taken as the multiplication (see [7]). There exists an isomorphism of Banach algebras between $\mathcal{M}(H)$ and $\mathcal{F}(B)$ (see [11; Proposition 2.1].).

Throughout this paper, let $\mathbb{R}$ and $\mathbb{C}$ denote the real numbers and the complex numbers, respectively, and put $\mathbb{C}_+ = \{z \in \mathbb{C} : \mathrm{Re}(z) > 0\}$ and $\widetilde{\mathbb{C}}_+ = \{z \in \mathbb{C} : z \neq 0, \mathrm{Re}(z) \geq 0\}$, where $\mathrm{Re}(z)$ means the real part of the complex number z.

Let F be a complex-valued scale-invariant measurable functional on the abstract Wiener space (B, H, ω) such that the Wiener integral

$$J(F;\lambda) = \int_B F(\lambda^{-1/2}x)\,d\omega(x)$$

exists as a finite number for all $\lambda > 0$. If there exists an analytic function $J^*(F;z)$ of z in the half-plane $\mathbb{C}_+$ such that $J^*(F;\lambda) = J(F;\lambda)$ for all $\lambda > 0$, then we define this analytic extension $J^*(F;z)$ of $J(F;\lambda)$ to be the *analytic Wiener integral of F over B with parameter z* and we write

$$\int_B^{anw_z} F(x)\,d\omega(x) \equiv \mathcal{I}^{anw}(F;z) = J^*(F;z)$$

for all $z \in \mathbb{C}_+$.

Let q be a non-zero real number and F a functional on B such that the analytic Wiener integral $\mathcal{I}^{anw}(F;z)$ exists for all $z \in \mathbb{C}_+$. If the following limit exists, then we call it the *analytic Feynman integral of F over B with parameter q* and we write

$$\int_B^{anf_q} F(x)\,d\omega(x) \equiv \mathcal{I}^{anf}(F;q) = \lim_{z\to -iq} \mathcal{I}^{anw}(F;z),$$

where z approaches $-iq$ through $\mathbb{C}_+$.

DEFINITION 2.3. Let $1 < p \le 2$ and let $\{F_n\}$ and F be scale-invariant measurable functionals on the abstract Wiener space (B, H, ω) such that for each $\rho > 0$,

$$\lim_{n\to\infty} \int_B |F_n(\rho x) - F(\rho x)|^{p'}\,d\omega(x) = 0. \tag{2.4}$$

Then we write

$$\operatorname*{l.i.m.}_{n\to\infty}(w_s^{p'})(F_n) \approx F, \tag{2.5}$$

and we call F the *scale-invariant limit in the mean of order p'*, where p and p' are related by $1/p + 1/p' = 1$.

A similar definition is understood when n is replaced by the continuously varying parameter z.

Now we are ready to define an L_p analytic Fourier-Feynman transform $(1 \le p \le 2)$ on the Fresnel class $\mathcal{F}(B)$.

DEFINITION 2.4. For each $z \in \mathbb{C}_+$, we define a transform $\mathcal{F}_z(F)$ of a functional F on the abstract Wiener space (B, H, ω) as follows:

$$\big(\mathcal{F}_z(F)\big)(y) = \mathcal{I}^{anw}(F(\cdot + y); z)\,, \quad y \in B. \tag{2.6}$$

Let q be a non-zero real number. In case that $1 < p \le 2$, we define the L_p *analytic Fourier-Feynman transform* $\mathcal{F}_{(q;p)}(F)$ for a functional F on (B, H, w) by

$$(\mathcal{F}_{(q;p)}(F))(y) = \underset{z \to -iq}{\mathrm{l.i.m.}}(w_s^{p'})(\mathcal{F}_z(F))(y) \tag{2.7}$$

for s-a.e. $y \in B$, whenever this limit exists, where z approaches $-iq$ through $\mathbb{C}_+$.

Let q be a non-zero real number. In case that $p = 1$, we define the L_1 *analytic Fourier-Feynman transform* $\mathcal{F}_{(q;1)}(F)$ of F by

$$(\mathcal{F}_{(q;1)}(F))(y) = \lim_{z \to -iq}(\mathcal{F}_z(F))(y), \tag{2.8}$$

for s-a.e. $y \in B$, where z approaches $-iq$ through $\mathbb{C}_+$.

We note that for $1 \le p \le 2$, $\mathcal{F}_{(q;p)}(F)$ is defined only s-a.e.. We also note that if $\mathcal{F}_{(q;p)}(F)$ exists and if $F \approx G$, then $\mathcal{F}_{(q;p)}(G)$ exists and $\mathcal{F}_{(q;p)}(F) \approx \mathcal{F}_{(q;p)}(G)$.

We finish this section by giving the definition of the convolution product of two functionals on the abstract Wiener space (B, H, ω).

DEFINITION 2.5. Let F and G be two complex-valued functionals on the abstract Wiener space (B, H, ω). For each $z \in \widetilde{\mathbb{C}}_+$, we define their *convolution product* $(F * G)_z$ as follows :

In case that z belongs to $\mathbb{C}_+$,

$$(F * G)_z(y) = \mathcal{I}^{anw}\Big[F\Big(\frac{1}{\sqrt{2}}(y + \cdot)\Big)\, G\Big(\frac{1}{\sqrt{2}}(y - \cdot)\Big)\,;\, z\Big] \tag{2.9}$$

for $y \in B$, if it exists.

In case that $z = -iq$ ($q \in \mathbb{R} - \{0\}$),

$$(F * G)_q(y) = \mathcal{I}^{anf}\Big[F\Big(\frac{1}{\sqrt{2}}(y + \cdot)\Big)\, G\Big(\frac{1}{\sqrt{2}}(y - \cdot)\Big)\,;\, q\Big] \tag{2.10}$$

for $y \in B$, if it exists.

3. L_p Analytic Fourier-Feynman Transforms and Convolution

We begin this section by showing the existence of the L_p analytic Fourier-Feynman transform for every functional in the Fresnel class $\mathcal{F}(B)$.

THEOREM 3.1. *Let $F \in \mathcal{F}(B)$ be given by (2.3) and let $1 \le p \le 2$. Then the transform $\mathcal{F}_z(F)$ exists for all $z \in \mathbb{C}_+$, it belongs to $\mathcal{F}(B)$, and the following formula*

$$(3.1) \qquad \big(\mathcal{F}_z(F)\big)(y) = \int_H \exp\Big\{-\frac{1}{2z}|h|^2 + i(h,y)^\sim\Big\}\, df(h)$$

holds for s-a.e. $y \in B$, where f is in $\mathcal{M}(H)$.

Moreover, the L_p analytic Fourier-Feynman transform $\mathcal{F}_{(q;p)}(F)$ exists for all $q \in \mathbb{R} - \{0\}$, it belongs to $\mathcal{F}(B)$, and the following formula

$$(3.2) \qquad \big(\mathcal{F}_{(q;p)}(F)\big)(y) = \int_H \exp\Big\{-\frac{i}{2q}|h|^2 + i(h,y)^\sim\Big\}\, df(h)$$

holds for s-a.e. $y \in B$, where f is in $\mathcal{M}(H)$.

Proof. We shall first calculate the transform $\mathcal{F}_t(F)$ for $t > 0$. Using Fubini's Theorem and the following integral formula :

$$(3.3) \qquad \int_B \exp\{it(h,x)^\sim\}\, d\omega(x) = \exp\Big\{-\frac{t^2}{2}|h|^2\Big\}, \quad h \in H,\ t \in \mathbb{R},$$

we have, for each $t > 0$,

$$(3.4) \qquad \begin{aligned} \big(\mathcal{F}_t(F)\big)(y) &= \textstyle\int_B \int_H \exp\big\{i\big(h, \frac{x}{\sqrt{t}} + y\big)^\sim\big\}\, df(h)\, d\omega(x) \\ &= \textstyle\int_H \exp\big\{-\frac{1}{2t}|h|^2 + i(h,y)^\sim\big\}\, df(h) \end{aligned}$$

for s-a.e. $y \in B$.

By using Morera's Theorem, we can verify that the last expression of (3.4) is an analytic function of t throughout $\mathbb{C}_+$, and is a bounded continuous function of t throughout $\widetilde{\mathbb{C}}_+$ for all $y \in B$, because f is in $\mathcal{M}(H)$. Therefore the transform $\mathcal{F}_z(F)$ exists for all $z \in \mathbb{C}_+$, and finally we can show that (3.1) and (3.2) hold.

Finally we shall show that $\mathcal{F}_z(F)$ belongs to $\mathcal{F}(B)$ for every $z \in \mathbb{C}_+$. Let z be in $\mathbb{C}_+$ and define a set function $\eta : \mathcal{B}(H) \longrightarrow \mathbb{C}$ as follows :

$$\eta(E) = \int_E \exp\Big\{-\frac{1}{2z}|h|^2\Big\}\, df(h), \quad E \in \mathcal{B}(H),$$

where $\mathcal{B}(H)$ is the Borel σ-algebra of H. Then it is obvious that η belongs to the Banach algebra $\mathcal{M}(H)$. Moreover, (3.1) is expressed as follows :

$$(\mathcal{F}_z(F))(y) = \int_H \exp\{i(h,y)^\sim\}\, d\eta(h).$$

Hence $\mathcal{F}_z(F)$ belongs to $\mathcal{F}(B)$.

Similarly, we can show that $\mathcal{F}_{(q;p)}(F)$ belongs to $\mathcal{F}(B)$. □

Definition 3.2. Let F be a functional defined on the abstract Wiener space (B,H,ω) and define a transform $\mathcal{F}_t^{(n)}(F)(t>0)$ of F as follows :

$$\mathcal{F}_t^{(n)}(F) = \big(\underbrace{\mathcal{F}_t \circ \cdots \circ \mathcal{F}_t}_{n}\big)(F),$$

that is, $\mathcal{F}_t^{(n)}$ means the n-times composition of $\mathcal{F}_t$, where $\mathcal{F}_t$ is equal to $\mathcal{F}_z$ for $z>0$ in (2.6) of Definition 2.4, and n is a natural number.

Let $\mathcal{F}_z^{(n)}(F)$ be an analytic extension of $\mathcal{F}_t^{(n)}(F)$ as a function of $z \in \mathbb{C}_+$. In case that $1<p\le 2$, for each $q \in \mathbb{R}-\{0\}$, we define the *n-repeated L_p analytic Fourier-Feynman transform* $\mathcal{F}_{(q;p)}^{(n)}(F)$ of F by

$$\mathcal{F}_{(q;p)}^{(n)}(F) \approx \operatorname*{l.i.m.}_{z\to -iq}(w_s^{p'})(\mathcal{F}_z^{(n)}(F)), \tag{3.5}$$

where z approaches $-iq$ through $\mathbb{C}_+$.

In case that $p=1$, for each $q\in\mathbb{R}-\{0\}$, we define the *n-repeated L_1 analytic Fourier-Feynman transform* $\mathcal{F}_{(q;1)}^{(n)}(F)$ of F by

$$\mathcal{F}_{(q;1)}^{(n)}(F) \approx \lim_{z\to -iq}(\mathcal{F}_z^{(n)}(F)), \tag{3.6}$$

where z approaches $-iq$ through $\mathbb{C}_+$.

Note that $\mathcal{F}_z^{(0)}(F) \equiv F \equiv \mathcal{F}_{(q;p)}^{(0)}(F)$, and $\mathcal{F}_z^{(1)}(F) \equiv \mathcal{F}_z(F)$ and $\mathcal{F}_{(q;p)}^{(1)}(F) \equiv \mathcal{F}_{(q;p)}(F)$.

By using the mathematical induction and proceeding as in the proof of Theorem 3.1, we can obtain the following theorem.

Theorem 3.3. *Let $F\in\mathcal{F}(B)$ be given by (2.3) and let $1\le p\le 2$. Then the transform $\mathcal{F}_z^{(n)}(F)$ exists for all $z\in\mathbb{C}_+$, it belongs to $\mathcal{F}(B)$, and the following formula*

$$\big(\mathcal{F}_z^{(n)}(F)\big)(y) = \int_H \exp\Big\{-\frac{n}{2z}|h|^2 + i(h,y)^\sim\Big\}\, df(h) \tag{3.7}$$

holds for s-a.e. $y \in B$, where f is in $\mathcal{M}(H)$ and $n = 0, 1, 2, \cdots$.

Moreover, for each $q \in \mathbb{R} - \{0\}$, the n-repeated L_p analytic Fourier-Feynman transform $\mathcal{F}^{(n)}_{(q;p)}(F)$ exists, it belongs to $\mathcal{F}(B)$, and the following formula

$$\big(\mathcal{F}^{(n)}_{(q;p)}(F)\big)(y) = \int_H \exp\Big\{-\frac{i\,n}{2q}|h|^2 + i(h,y)^\sim\Big\}\, df(h) \tag{3.8}$$

holds for s-a.e. $y \in B$, where f is in $\mathcal{M}(H)$ and $n = 0, 1, 2, \cdots$.

Note that (3.7) and (3.8) are reduced to (3.1) and (3.2), respectively, if we take $n = 1$ in (3.7) and (3.8).

Theorem 3.4. *Let F and G be in $\mathcal{F}(B)$ which are given by (2.3). Then the convolution product $\big((\mathcal{F}^{(n)}_z F) * (\mathcal{F}^{(m)}_z G)\big)_z$ exists for each $z \in \mathbb{C}_+$, it belongs to $\mathcal{F}(B)$, and the following formula*

(3.9)

$$\begin{aligned}
&\big((\mathcal{F}^{(n)}_z F) * (\mathcal{F}^{(m)}_z G)\big)_z(y)\\
&= \textstyle\int_{H^2} \exp\big\{-\frac{n}{2z}|u|^2 - \frac{m}{2z}|v|^2 - \frac{1}{4z}|u-v|^2 + \frac{i}{\sqrt{2}}(u+v,y)^\sim\big\}\, df(u)\, dg(v)
\end{aligned}$$

holds for s-a.e. $y \in B$, where f and g are in $\mathcal{M}(H)$ and $m, n = 0, 1, 2, \cdots$.

Furthermore, the convolution product $\big((\mathcal{F}^{(n)}_{(q;p)} F) * (\mathcal{F}^{(m)}_{(q;p)} G)\big)_q$ exists for every $q \in \mathbb{R} - \{0\}$, it belongs to $\mathcal{F}(B)$, and it is given by

(3.10)

$$\begin{aligned}
&\big((\mathcal{F}^{(n)}_{(q;p)} F) * (\mathcal{F}^{(m)}_{(q;p)} G)\big)_q(y)\\
&= \textstyle\int_{H^2} \exp\big\{-\frac{i\,n}{2q}|u|^2 - \frac{i\,m}{2q}|v|^2 - \frac{i}{4q}|u-v|^2 + \frac{i}{\sqrt{2}}(u+v,y)^\sim\big\}\, df(u)\, dg(v)\,,
\end{aligned}$$

for s-a.e. $y \in B$, where f and g are in $\mathcal{M}(H)$ and $m, n = 0, 1, 2, \cdots$.

Proof. By using Fubini's Theorem, Definition 2.5, (3.3), and (3.7), we first calculate $\big((\mathcal{F}^{(n)}_t F) * (\mathcal{F}^{(m)}_t G)\big)_t$ for every $t > 0$ as follows :

$$\begin{aligned}
&\big((\mathcal{F}^{(n)}_t F) * (\mathcal{F}^{(m)}_t G)\big)_t(y)\\
&= \textstyle\int_B \big(\mathcal{F}^{(n)}_t F\big)\big(\frac{1}{\sqrt{2}}\big(y + \frac{x}{\sqrt{t}}\big)\big)\big(\mathcal{F}^{(m)}_t G\big)\big(\frac{1}{\sqrt{2}}\big(y - \frac{x}{\sqrt{t}}\big)\big)\, d\omega(x)\\
&= \textstyle\int_{H^2} \exp\big\{-\frac{n}{2t}|u|^2 - \frac{m}{2t}|v|^2 + \frac{i}{\sqrt{2}}(u+v,y)^\sim\big\}\\
&\quad \cdot\big[\textstyle\int_B \exp\big\{\frac{i}{\sqrt{2t}}(u-v,x)^\sim\big\}\, d\omega(x)\big]\, df(u)\, dg(v)\\
&= \textstyle\int_{H^2} \exp\big\{-\frac{n}{2t}|u|^2 - \frac{m}{2t}|v|^2 + \frac{i}{\sqrt{2}}(u+v,y)^\sim - \frac{1}{4t}|u-v|^2\big\}\, df(u)\, dg(v).
\end{aligned}$$

By using Morera's Theorem, we can verify that the last expression is an analytic function of t throughout $\mathbb{C}_+$, and is a bounded continuous function of t over $\widetilde{\mathbb{C}}_+$ for all y in B, because f and g are in $\mathcal{M}(H)$. Therefore, we can show that (3.9) and (3.10) hold.

Next we shall show that $\big((\mathcal{F}_z^{(n)}F) * (\mathcal{F}_z^{(m)}G)\big)_z$ belongs to $\mathcal{F}(B)$ for every $z \in \mathbb{C}_+$. Let z be in $\mathbb{C}_+$ and define a set function $\nu : \mathcal{B}(H^2) \longrightarrow \mathbb{C}$ by

$$\nu(E) = \int_E \exp\Big\{-\frac{n}{2z}|u|^2 - \frac{m}{2z}|v|^2 - \frac{1}{4z}|u-v|^2\Big\}\, df(u)\, dg(v), \ E \in \mathcal{B}(H^2).$$

Then ν is a complex-valued countably additive Borel measure on $\mathcal{B}(H^2)$.

Now define a function $\varphi : H^2 \longrightarrow H$ as follows :

$$\varphi(u,v) = \frac{1}{\sqrt{2}}(u+v), \quad (u,v) \in H^2.$$

Then φ is continuous , and so it is a Borel measurable function. Hence $\mu = \nu \cdot \varphi^{-1}$ belongs to the Banach algebra $\mathcal{M}(H)$. By using the Change of Variable Formula, we have

$$\big((\mathcal{F}_z^{(n)}F) * (\mathcal{F}_z^{(m)}G)\big)_z(y) = \int_H \exp\big\{i(w,y)^\sim\big\}\, d\mu(w).$$

Hence $\big((\mathcal{F}_z^{(n)}F) * (\mathcal{F}_z^{(m)}G)\big)_z$ belongs to $\mathcal{F}(B)$.

Similarly, we can show that $\big((\mathcal{F}_{(q;p)}^{(n)}F) * (\mathcal{F}_{(q;p)}^{(m)}G)\big)_q$ belongs to $\mathcal{F}(B)$. $\square$

Our next theorem shows that the L_p analytic Fourier-Feynman transform of the convolution product for two functionals in the Fresnel class $\mathcal{F}(B)$ is a product of Fourier-Feynman transforms for each functional.

Theorem 3.5. *Let F and G be as in Theorem 3.4 and let $1 \le p \le 2$. Then the transform $\mathcal{F}_z\big((\mathcal{F}_z^{(n)}F) * (\mathcal{F}_z^{(m)}G)\big)_z$ exists for all $z \in \mathbb{C}_+$, and the following formula*

(3.11)
$$\big(\mathcal{F}_z((\mathcal{F}_z^{(n)}F) * (\mathcal{F}_z^{(m)}G))_z\big)(y) = \big(\mathcal{F}_z^{(n+1)}F\big)\Big(\frac{y}{\sqrt{2}}\Big)\,\big(\mathcal{F}_z^{(m+1)}G\big)\Big(\frac{y}{\sqrt{2}}\Big)$$

holds for s-a.e. $y \in B$, where $m, n = 0, 1, 2, \cdots$.

Furthermore, for each $q \in \mathbb{R} - \{0\}$, L_p analytic Fourier-Feynman transform $\mathcal{F}_{(q;p)}\big((\mathcal{F}_{(q;p)}^{(n)}F) * (\mathcal{F}_{(q;p)}^{(m)}G)\big)_q$ is given by

(3.12)
$$\big(\mathcal{F}_{(q;p)}((\mathcal{F}_{(q;p)}^{(n)}F) * (\mathcal{F}_{(q;p)}^{(m)}G))_q\big)(y) = \big(\mathcal{F}_{(q;p)}^{(n+1)}F\big)\Big(\frac{y}{\sqrt{2}}\Big)\big(\mathcal{F}_{(q;p)}^{(m+1)}G\big)\Big(\frac{y}{\sqrt{2}}\Big),$$

where $m, n = 0, 1, 2, \ldots$.

Proof. By using Fubini's Theorem, (3.3) and (3.9), we first calculate the transform $\mathcal{F}_t\big((\mathcal{F}_t^{(n)}F) * (\mathcal{F}_t^{(m)}G)\big)_t$ for all $t > 0$ as follows:

(3.13)
$$\begin{aligned}
&\big(\mathcal{F}_t((\mathcal{F}_t^{(n)}F) * (\mathcal{F}_t^{(m)}G))_t\big)(y)\\
&= \int_B \big((\mathcal{F}_t^{(n)}F) * (\mathcal{F}_t^{(m)}G)\big)_t\big(\tfrac{x}{\sqrt{t}} + y\big)\, d\omega(x)\\
&= \int_{H^2} \exp\big\{-\tfrac{n}{2t}|u|^2 - \tfrac{m}{2t}|v|^2 - \tfrac{1}{4t}|u-v|^2 + \tfrac{i}{\sqrt{2}}(u+v, y)^{\sim}\big\}\\
&\quad \cdot \big[\int_B \exp\big\{\tfrac{i}{\sqrt{2t}}(u+v, x)^{\sim}\big\}\, d\omega(x)\big]\, df(u)\, dg(v)\\
&= \int_{H^2} \exp\big\{-\tfrac{(n+1)}{2t}|u|^2 - \tfrac{(m+1)}{2t}|v|^2 + \tfrac{i}{\sqrt{2}}(u+v, y)^{\sim}\big\} df(u) dg(v).
\end{aligned}$$

By using Morera's Theorem, we can verify that the last expression in (3.13) is an analytic function of t throughout $\mathbb{C}_+$, and is a bounded continuous function of t over $\widetilde{\mathbb{C}}_+$ for all $y \in B$, because f and g are in $\mathcal{M}(H)$. Therefore, for each $z \in \mathbb{C}_+$, the following formula

(3.14)
$$\begin{aligned}
&\big(\mathcal{F}_z((\mathcal{F}_z^{(n)}F) * (\mathcal{F}_z^{(m)}G))_z\big)(y)\\
&= \int_{H^2} \exp\big\{-\tfrac{(n+1)}{2z}|u|^2 - \tfrac{(m+1)}{2z}|v|^2 + \tfrac{i}{\sqrt{2}}(u+v, y)^{\sim}\big\}\, df(u)\, dg(v)
\end{aligned}$$

holds for s-a.e. $y \in B$.

On the other hand, using (3.7), we can show that for every $z \in \mathbb{C}_+$, the following formula

(3.15)
$$\begin{aligned}
&\big(\mathcal{F}_z^{(n+1)}F\big)\big(\tfrac{y}{\sqrt{2}}\big)\big(\mathcal{F}_z^{(m+1)}G\big)\big(\tfrac{y}{\sqrt{2}}\big)\\
&= \int_{H^2} \exp\big\{-\tfrac{(n+1)}{2z}|u|^2 - \tfrac{(m+1)}{2z}|v|^2 + \tfrac{i}{\sqrt{2}}(u+v, y)^{\sim}\big\} df(u) dg(v)
\end{aligned}$$

holds for s-a.e. $y \in B$.

Therefore, (3.11) follows from (3.14) and (3.15), and finally (3.12) comes from (3.11) with the help of Definition 2.3. □

Our next theorem shows that an interesting Parseval's identity holds on the Fresnel class $\mathcal{F}(B)$.

THEOREM 3.6. *Let F and G be as in Theorem* 3.4 *and let* $1 \le p \le 2$. *Then for each $q \in \mathbb{R} - \{0\}$, the following Parseval's identity holds* :

$$\begin{aligned}
&\mathcal{F}_{(-q;p)}\big(\mathcal{F}_{(q;p)}((\mathcal{F}_{(q;p)}^{(n)}F) * (\mathcal{F}_{(q;p)}^{(m)}G))_q\big)(0)\\
&= \mathcal{F}_{(q;p)}\big(\big(\mathcal{F}_{(q;p)}^{(n)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big(\mathcal{F}_{(q;p)}^{(m)}G\big)\big(-\tfrac{\cdot}{\sqrt{2}}\big)\big)(0)\,,
\end{aligned} \tag{3.16}$$

where $m,\ n = 0, 1, 2, \cdots$.

Proof. We first calculate the transform $\mathcal{F}_t\big(\mathcal{F}_{(q;p)}((\mathcal{F}^{(n)}_{(q;p)}F)*(\mathcal{F}^{(m)}_{(q;p)}G))_q\big)$ for all $t>0$, where q is a non-zero real number. Using Fubini's Theorem, (3.3), (3.8), and (3.12), we have, for all $t>0$,

$$\begin{aligned}
&\mathcal{F}_t\big(\mathcal{F}_{(q;p)}((\mathcal{F}^{(n)}_{(q;p)}F)*(\mathcal{F}^{(m)}_{(q;p)}G))_q\big)(0)\\
&=\mathcal{F}_t\big(\big(\mathcal{F}^{(n+1)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big(\mathcal{F}^{(m+1)}_{(q;p)}G\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big)(0)\\
&=\int_B\big(\mathcal{F}^{(n+1)}_{(q;p)}F\big)\big(\tfrac{x}{\sqrt{2t}}\big)\big(\mathcal{F}^{(m+1)}_{(q;p)}G\big)\big(\tfrac{x}{\sqrt{2t}}\big)\,d\omega(x)\\
&=\int_{H^2}\exp\big\{-\tfrac{i(n+1)}{2q}|u|^2-\tfrac{i(m+1)}{2q}|v|^2)\big\}\big[\int_B\exp\big\{\tfrac{i}{\sqrt{2t}}(u+v,x)^\sim\big\}\,d\omega(x)\big]\\
&\quad\cdot df(u)\,dg(v)\\
&=\int_{H^2}\exp\big\{-\tfrac{i(n+1)}{2q}|u|^2-\tfrac{i(m+1)}{2q}|v|^2-\tfrac{1}{4t}|u+v|^2\big\}\,df(u)\,dg(v).
\end{aligned}$$

Since the last expression has an analytic extension for t over $\mathbb{C}_+$, and is a bounded continuous function of t over $\widetilde{\mathbb{C}}_+$, we can show that the following formula

$$\begin{aligned}
&\mathcal{F}_{(-q;p)}\big(\mathcal{F}_{(q;p)}((\mathcal{F}^{(n)}_{(q;p)}F)*(\mathcal{F}^{(m)}_{(q;p)}G))_q\big)(0)\\
&=\underset{t\to iq}{\mathrm{l.i.m}}(w_s^{p'})\mathcal{F}_t\big(\mathcal{F}_{(q;p)}((\mathcal{F}^{(n)}_{(q;p)}F)*(\mathcal{F}^{(m)}_{(q;p)}G))_q\big)(0)\\
&=\int_{H^2}\exp\big\{-\tfrac{i\,n}{2q}|u|^2-\tfrac{i\,m}{2q}|v|^2-\tfrac{i}{4q}|u-v|^2\big\}\,df(u)\,dg(v)
\end{aligned}\tag{3.17}$$

holds.

Next we calculate the transform $\mathcal{F}_t\big((\mathcal{F}^{(n)}_{(q;p)}F)(\frac{\cdot}{\sqrt{2}})(\mathcal{F}^{(m)}_{(q;p)}G)(-\frac{\cdot}{\sqrt{2}})\big)(0)$ for all $t>0$. Using (3.3) and Fubini's Theorem, we obtain the following formula

$$\begin{aligned}
&\mathcal{F}_t\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\,\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big(-\tfrac{\cdot}{\sqrt{2}}\big)\big)(0)\\
&=\int_B\big(\mathcal{F}^{(n)}_{(q;p)}F\big)\big(\tfrac{x}{\sqrt{2t}}\big)\,\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big(-\tfrac{x}{\sqrt{2t}}\big)\,d\omega(x)\\
&=\int_{H^2}\exp\big\{-\tfrac{i\,n}{2q}|u|^2-\tfrac{i\,m}{2q}|v|^2\big\}\big[\int_B\exp\big\{\tfrac{i}{\sqrt{2t}}(u-v,x)^\sim\big\}d\omega(x)\big]df(u)\,dg(v)\\
&=\int_{H^2}\exp\big\{-\tfrac{i\,n}{2q}|u|^2-\tfrac{i\,m}{2q}|v|^2-\tfrac{1}{4t}|u-v|^2\big\}\,df(u)\,dg(v).
\end{aligned}$$

Since the last expression has an analytic extension for t over $\mathbb{C}_+$, and is a bounded continuous function of t throughout $\widetilde{\mathbb{C}}_+$, we can show that the following formula

$$\begin{aligned}
&\mathcal{F}_{(q;p)}\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\,\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big(-\tfrac{\cdot}{\sqrt{2}}\big)\big)(0)\\
&=\underset{t\to -iq}{\mathrm{l.i.m}}(w_s^{p'})\mathcal{F}_t\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\,\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big(-\tfrac{\cdot}{\sqrt{2}}\big)\big)(0)\\
&=\int_{H^2}\exp\big\{-\tfrac{i\,n}{2q}|u|^2-\tfrac{i\,m}{2q}|v|^2-\tfrac{i}{4q}|u-v|^2\big\}\,df(u)\,dg(v)
\end{aligned}\tag{3.18}$$

holds.

Therefore, (3.16) comes from (3.17) and (3.18). $\square$

THEOREM 3.7. *Let F and G be as in Theorem 3.4 and let $1 \le p \le 2$. Then for each non-zero real number q, the following formula*

(3.19)
$$\Big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big)*\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\Big)_{-q}(y) = \mathcal{F}_{(q;p)}\Big(\big(\mathcal{F}^{(n-1)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big(\mathcal{F}^{(m-1)}_{(q;p)}G\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\Big)(y)$$

holds for s-a.e. $y \in B$, where $m, n = 1, 2, 3, \cdots$.

Proof. Let q be any non-zero real number. Using (3.3), (3.8) and Fubini's Theorem, for each $t > 0$ we first calculate the expression $\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big)*\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big)_t(y)$ for each $y \in B$ as follows

(3.20)
$$\begin{aligned}
&\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big) * \big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big)_t(y)\\
&= \int_B \big(\mathcal{F}^{(n)}_{(q;p)}F\big)\big(\tfrac{1}{\sqrt{2}}\big(y + \tfrac{x}{\sqrt{t}}\big)\big)\,\big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big(\tfrac{1}{\sqrt{2}}\big(y - \tfrac{x}{\sqrt{t}}\big)\big)\,d\omega(x)\\
&= \int_{H^2} \exp\big\{\tfrac{i}{\sqrt{2}}(u+v,y)^\sim - \tfrac{i\,n}{2q}|u|^2 - \tfrac{i\,m}{2q}|v|^2\big\}\big[\int_B \exp\big\{\tfrac{i}{\sqrt{2t}}(u-v,x)^\sim\big\}\\
&\quad\cdot d\omega(x)\big]\,df(u)\,dg(v)\\
&= \int_{H^2} \exp\big\{\tfrac{i}{\sqrt{2}}(u+v,y)^\sim - \tfrac{i\,n}{2q}|u|^2 - \tfrac{i\,m}{2q}|v|^2 - \tfrac{1}{4t}|u-v|^2\big\}\,df(u)\,dg(v).
\end{aligned}$$

By using Morera's Theorem, we can verify that the last expression in (3.20) is an analytic function of t throughout $\mathbb{C}_+$, and is a bounded continuous function of t throughout $\widetilde{\mathbb{C}}_+$. Therefore, for each non-zero real number q, we have the following formula

(3.21)
$$\begin{aligned}
&\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big) * \big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big)_{-q}(y)\\
&= \underset{t\to iq}{\mathrm{l.\,i.\,m}}(w_s^{p'})\big(\big(\mathcal{F}^{(n)}_{(q;p)}F\big) * \big(\mathcal{F}^{(m)}_{(q;p)}G\big)\big)_t(y)\\
&= \int_{H^2} \exp\big\{\tfrac{i}{\sqrt{2}}(u+v,y)^\sim - \tfrac{i\,n}{2q}|u|^2 - \tfrac{i\,m}{2q}|v|^2 + \tfrac{i}{4q}|u-v|^2\big\}\,df(u)dg(v).
\end{aligned}$$

for s-a.e $y \in B$.

Next, for each $t > 0$, we obtain the following formula

(3.22)
$$\begin{aligned}
&\mathcal{F}_t\big(\big(\mathcal{F}^{(n-1)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big(\mathcal{F}^{(m-1)}_{(q;p)}G\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big)(y)\\
&= \int_B \big(\mathcal{F}^{(n-1)}_{(q;p)}F\big)\big(\tfrac{1}{\sqrt{2}}\big(\tfrac{x}{\sqrt{t}} + y\big)\big)\big(\mathcal{F}^{(m-1)}_{(q;p)}G\big)\big(\tfrac{1}{\sqrt{2}}\big(\tfrac{x}{\sqrt{t}} + y\big)\big)\,d\omega(x)\\
&= \int_{H^2} \exp\big\{\tfrac{i}{\sqrt{2}}(u+v,y)^\sim - \tfrac{i(n-1)}{2q}|u|^2 - \tfrac{i(m-1)}{2q}|v|^2\big\}\\
&\quad\cdot\big[\int_B \exp\big\{\tfrac{i}{\sqrt{2t}}(u+v,x)^\sim\big\}\,d\omega(x)\big]\,df(u)dg(v)\\
&= \int_{H^2} \exp\big\{\tfrac{i}{\sqrt{2}}(u+v,y)^\sim - \tfrac{i(n-1)}{2q}|u|^2 - \tfrac{i(m-1)}{2q}|v|^2 - \tfrac{1}{4t}|u+v|^2\big\}\\
&\quad\cdot df(u)\,dg(v)
\end{aligned}$$

for s-a.e. $y \in B$.

By using Morera's Theorem, we can verify that the last expression in (3.22) is an analytic function of t throughout $\mathbb{C}_+$, and is a bounded continuous function of t throughout $\widetilde{\mathbb{C}}_+$. Therefore, for each non-zero real number, we have the following formula

(3.23)

$$\begin{aligned}
&\mathcal{F}_{(q;p)}\big(\big(\mathcal{F}^{(n-1)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big(\mathcal{F}^{(m-1)}_{(q;p)}G\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big)(y)\\
&= \underset{t\to -iq}{\mathrm{l.\,i.\,m}}(w_s^{p'})\mathcal{F}_t\big(\big(\mathcal{F}^{(n-1)}_{(q;p)}F\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big(\mathcal{F}^{(m-1)}_{(q;p)}G\big)\big(\tfrac{\cdot}{\sqrt{2}}\big)\big)(y)\\
&= \int_{H^2}\exp\big\{\tfrac{i}{\sqrt{2}}(u+v,y)^\sim - \tfrac{i\,n}{2q}|u|^2 - \tfrac{i\,m}{2q}|v|^2 + \tfrac{i}{4q}|u-v|^2\big\}df(u)dg(v)
\end{aligned}$$

for s-a.e. $y \in B$.

Therefore, (3.19) comes from (3.21) and (3.23). $\square$

4. Corollaries

In this section we apply our results in the preceding section to the classical Wiener space to obtain several results in [9] as corollaries.

Fix $T > 0$ and let $B_o \equiv C_o[0,T]$ be the real separable Banach space of all real-valued continuous functions f on the closed interval $[0,T]$ which vanish at 0 and equip B_o with the uniform norm. Let $(B_o, \mathcal{W}(B_o), m_w)$ be the classical Wiener space , where m_w is the Wiener measure on the σ-algebra $\mathcal{W}(B_o)$ which is the completion of Borel σ-algebra $\mathcal{B}(B_o)$. Let $H_o \equiv H_o[0,T]$ be the space of all real-valued functions f on [0,T] which are absolutely continuous and vanish at 0, and whose derivative Df is in $L_2[0,T]$.

Define an inner product $\langle\cdot,\cdot\rangle$ on H_o as follows :

$$\langle f, g\rangle = \int_0^T (Df)(s)\,(Dg)(s)\,ds, \quad f,\, g \in H_o.$$

Then H_o is a real separable infinite dimensional Hilbert space, and (B_o, H_o, m_w) is a typical example of an abstract Wiener space (see [13]). It is well known [11] that for each $h \in H_o$,

$$(h,x)^\sim = \int_0^T (Dh)(s)\,\widetilde{d}x(s)$$

holds for s-a.e. $x \in H_o$, where $\int_0^T (Dh)(s)\,\widetilde{d}x(s)$ is the Paley-Wiener-Zygmund stochastic integral of Dh (see [4]).

In [4], Cameron and Storvick introduced a Banach algebra $\mathcal{S}$ of functionals on B_o given as follows :

$$\mathcal{S} = \Big\{ F : F(x) = \int_{L_2[0,T]} \exp\Big\{ i \int_0^T v(s)\widetilde{d}x(s) \Big\}\, df(v),\ f \in \mathcal{M}(L_2[0,T]) \Big\}.$$

Let I be the unitary operator from $L_2[0,T]$ onto H_o given by

$$Iv(t) = \int_0^t v(s)\, ds, \qquad \text{for } v \in L_2[0,T] \text{ and } t \in [0,T].$$

If

$$F(x) = \int_{L_2[0,T]} \exp\Big\{ i \int_0^T v(s)\, \widetilde{d}x(s) \Big\}\, df(v) \tag{4.1}$$

for some $f \in \mathcal{M}(L_2[0,T])$, then we have

$$F(x) = \int_{H_o} \exp\big\{ i(h,x)^{\sim} \big\}\, d(f \circ I^{-1})(h).$$

Conversely, if

$$F(x) = \int_{H_o} \exp\big\{ i(h,x)^{\sim} \big\}\, df(h)$$

for some $f \in \mathcal{M}(H_o)$, then we have

$$F(x) = \int_{L_2[0,T]} \exp\Big\{ i \int_0^T v(s)\, \widetilde{d}x(s) \Big\}\, d(f \circ I)(v).$$

Thus we show that $F \in \mathcal{S}$ if and only if $F \in \mathcal{F}(B_o)$ (see [11]).

Corollary 4.1. (*Theorem* 3.1 *in* [9]) *Let* $F \in \mathcal{S}$ *be given by* (4.1) *and let* $1 \le p \le 2$. *Then the* L_p *analytic Fourier-Feynman transform* $T_q^{(p)}(F)$ *exists for all* $q \in \mathbb{R} - \{0\}$, *and the following formula*

$$\big(T_q^{(p)}(F)\big)(y) = \int_{L_2[0,T]} \exp\Big\{ i \int_0^T v(t)\, \widetilde{d}y(t) - \frac{i}{2q} \int_0^T v^2(t)dt \Big\}\, df(v) \tag{4.2}$$

holds for s-a.e. $y \in B_o$.

Proof. In (3.2) of Theorem 3.1, taking $H = H_o$, $\mathcal{F}_{(q;p)}(F) = T_q^{(p)}F$, and $h = Iv$ for some $v \in L_2[0,T]$, we have the desired result. □

Corollary 4.2. (*Theorem* 3.2 *in* [9]) *Let F and G be elements of $\mathcal{S}$ with corresponding complex Borel measures f and g in $\mathcal{M}(L_2[0,T])$. Then the convolution product $(F*G)_q$ exists for all $q \in \mathbb{R} - \{0\}$, and the following formula*

$$(4.3)\qquad \begin{aligned}(F*G)_q(y) &= \int_{L_2^2[0,T]} \exp\Big\{\tfrac{i}{\sqrt{2}}\int_0^T (v(t)+w(t))\,\widetilde{d}y(t)\Big\}\\ &\quad\cdot \exp\Big\{-\tfrac{i}{4q}\int_0^T (v(t)-w(t))^2\,dt\Big\}\,df(v)\,dg(w)\end{aligned}$$

holds for s-a.e. $y \in B_o$.

Proof. In (3.10) of Theorem 3.4, taking $m=n=0$, $H=H_o$, $u=Iv$ and $v=Iw$ for some v and w in $L_2[0,T]$, we have the desired result. □

Corollary 4.3. (*Theorem* 3.3 *in* [9]) *Let F and G be as in Corollary 4.2. Then, for all $q \in \mathbb{R} - \{0\}$, the following formula*

$$(4.4)\qquad \big(T_q^{(p)}(F*G)_q\big)(y) = \big(T_q^{(p)}(F)\big)\big(y/\sqrt{2}\big)\big(T_q^{(p)}(G)\big)\big(y/\sqrt{2}\big)$$

holds for s-a.e. $y \in B_o$ *where* $1 \le p \le 2$.

Proof. In (3.12) of Theorem 3.5, taking $m=n=0$ and $\mathcal{F}_{(q;p)}(F) \equiv \mathcal{F}^{(1)}_{(q;p)}(F) = T_q^{(p)}(F)$ for every $F \in \mathcal{S}$, , we have the desired result. □

Corollary 4.4. (*Theorem* 3.4 *in* [9]) *Let F and G be as in Corollary 4.2. Then, for all $q \in \mathbb{R} - \{0\}$, the Parseval's identity*

$$(4.5)\qquad \begin{aligned}&\int_{C_o[0,T]}^{anf_{-q}} \big(T_q^{(p)}(F*G)_q\big)(x)\,m_w(dx)\\ &= \int_{C_o[0,T]}^{anf_q} F(x/\sqrt{2})G(-x/\sqrt{2})\,m_w(dx)\end{aligned}$$

holds where $1 \le p \le 2$.

Proof. In (3.16) of Theorem 3.6, taking $m=n=0$,

$$(\mathcal{F}_{(q;p)}(F))(0) = \int_{C_o[0,T]}^{anf_q} F(x)\,m_w(dx), \text{ and } \mathcal{F}_{(q;p)}(F) = T_q^{(p)}(F)$$

for every $F \in \mathcal{S}$, we have the desired result. □

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